Novel topologies in superconducting junctions

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Overview of 3 lectures

- Lecture #1
  - Basics of superconducting junctions and structures
  - ‘Old’ topology – Majorana
- Lecture #2
  - Weyl topology
  - Basics of semiclassical description: circuit theory
- Lecture #3
  - Semiclassical topology
  - Smiling gaps topology

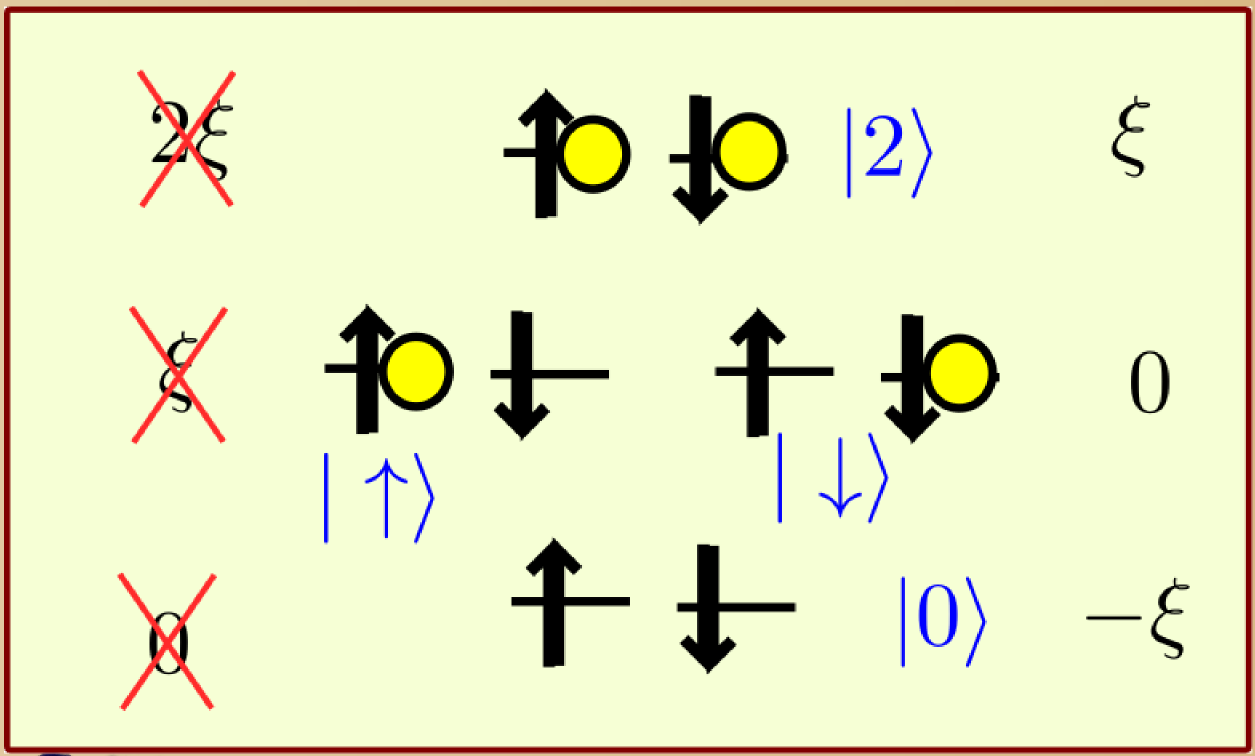
Lecture #1

- Quantum states in superconductors
- Scattering approach
- Andreev reflection
- Beenakker formula
- Practical matters
- Majorana: idea and status
- Majorana: simple example and consequencies

Quantum states for fermions

- 1 level x 2 spin directions = 4 states

$$\hat{H} = \sum_{levels} \epsilon_l \left(\hat{a}^\dagger_\uparrow \hat{a}_\uparrow + \hat{a}^\dagger_\downarrow \hat{a}_\downarrow\right) - \mu \hat{N}$$



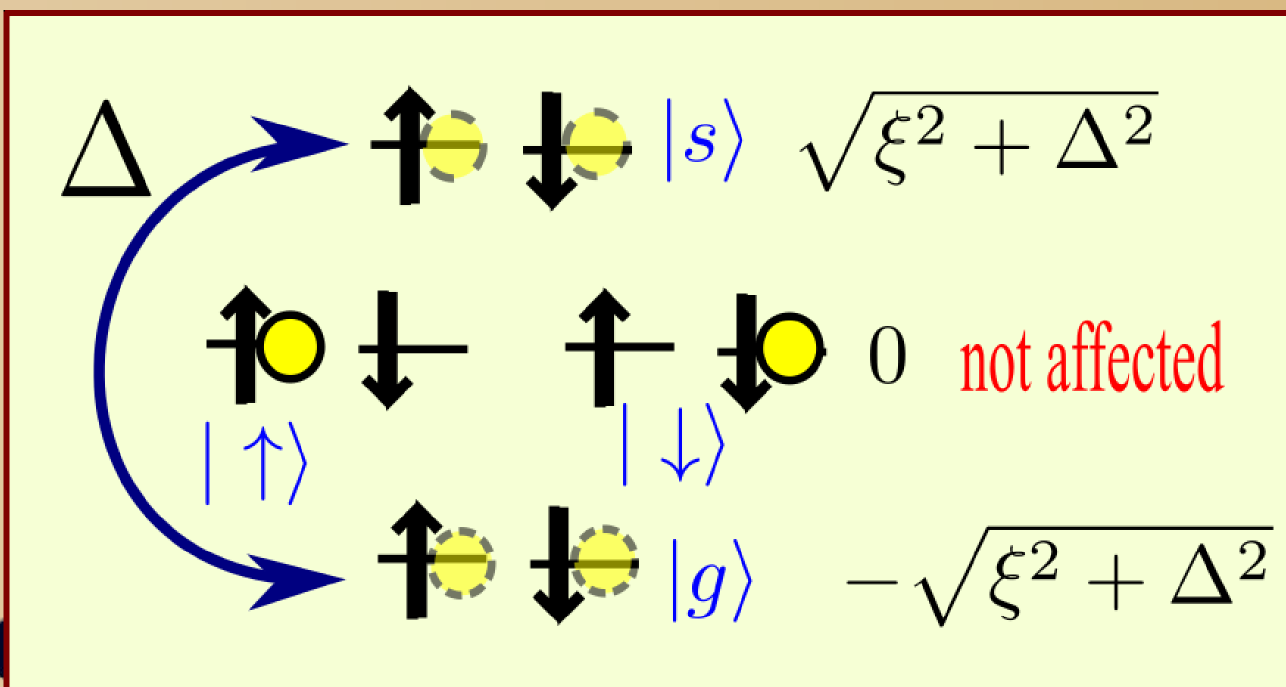
$$\xi = \epsilon - \mu$$

$$\boxed{E_g = -\frac{1}{2} \sum E_{ex}}$$

Quantum states in superconductors

- Superconducting condensate

$$\hat{H} = \sum_{levels} \epsilon_l \left(\hat{a}_\uparrow^\dagger \hat{a}_\uparrow + \hat{a}_\downarrow^\dagger \hat{a}_\downarrow \right) - \mu \hat{N} + \Delta \hat{a}_\uparrow^\dagger \hat{a}_\downarrow^\dagger + \Delta^* \hat{a}_\downarrow \hat{a}_\uparrow$$



$|s\rangle \quad |g\rangle$

Mixtures of

$|0\rangle \quad |2\rangle$

$$\boxed{E_g = -\frac{1}{2} \sum E_{ex}}$$

BdG Hamiltonian

- Bogoluybov transform cr/ann operators for the excitations
- It doubles the basis!

$$\hat{\Delta} \rightarrow i\sigma_y \hat{\Delta}$$

$$\mathcal{H}_{\mathrm{BdG}} \equiv \begin{bmatrix} \hat{H} & \hat{\Delta} \\ \hat{\Delta}^\dagger & -\hat{H}^T \end{bmatrix}$$

$$\bar{\mathcal{H}}_{\mathrm{BdG}} = \begin{bmatrix} \hat{H} & -i\hat{\Delta} \\ i\hat{\Delta}^\dagger & \hat{\bar{H}} \end{bmatrix}$$

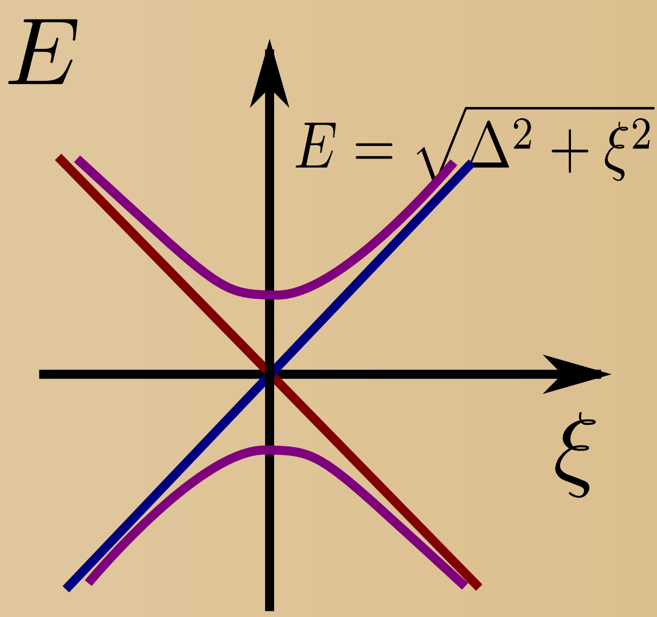
- Mirror symmetry of eigenvalues
- Only positive ones matter

$$\pm E$$

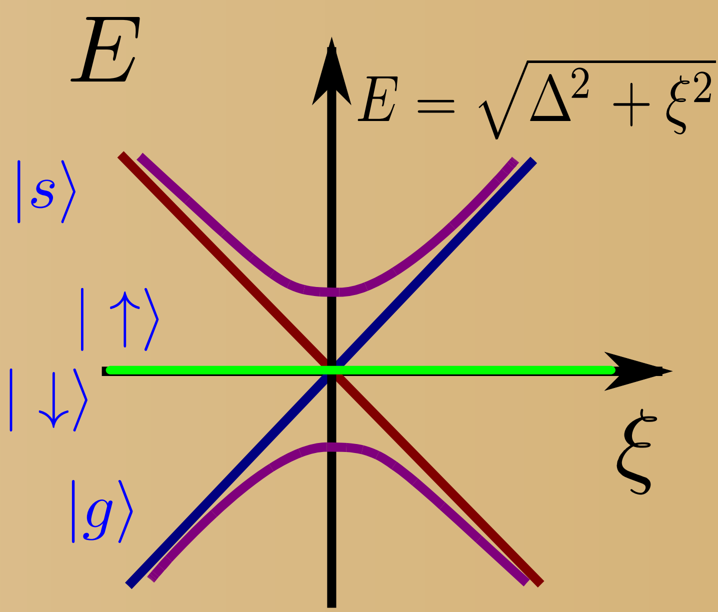
The most confusing slide :)

$$\pm E$$

- BdG spectrum

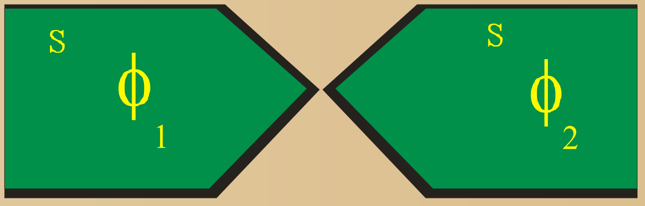


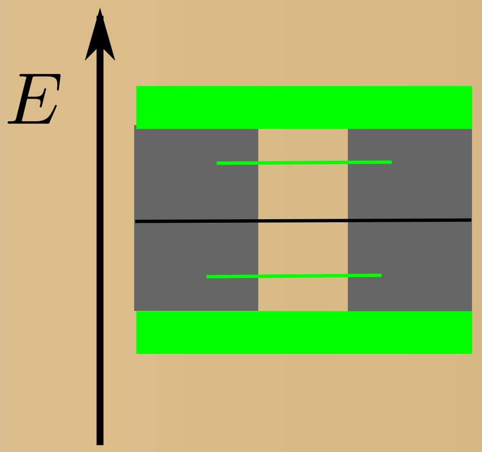
- Energies of the states



Gap in the bulk

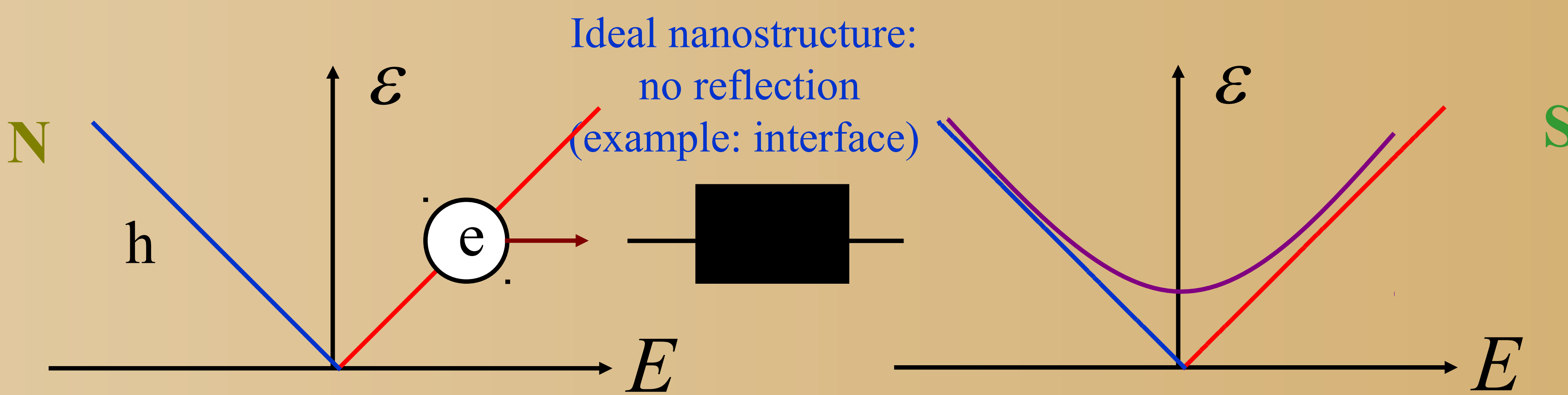
- Old news?
- Strictly no excitations
- Enables quantum states in nanostructures
  - Superconducting qubits
  - Andreev bound states





Andreev reflection

How to transport? (Given energy conservation)



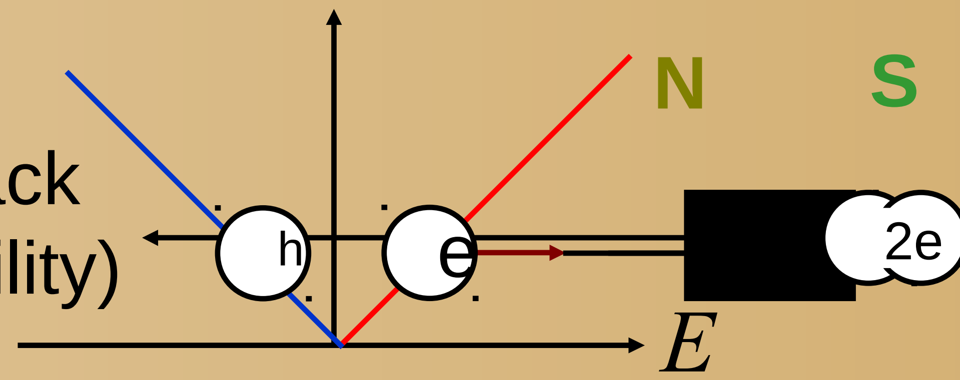
$\varepsilon < \Delta$: No states in superconductor. Andreev reflection

$\varepsilon > \Delta$: Electron => quasiparticle, still partially Andreev reflected.

Andreev reflection

Ideal nanostructure $\varepsilon < \Delta$

An electron is reflected back as a hole (the only possibility)



Let us check conservation laws:

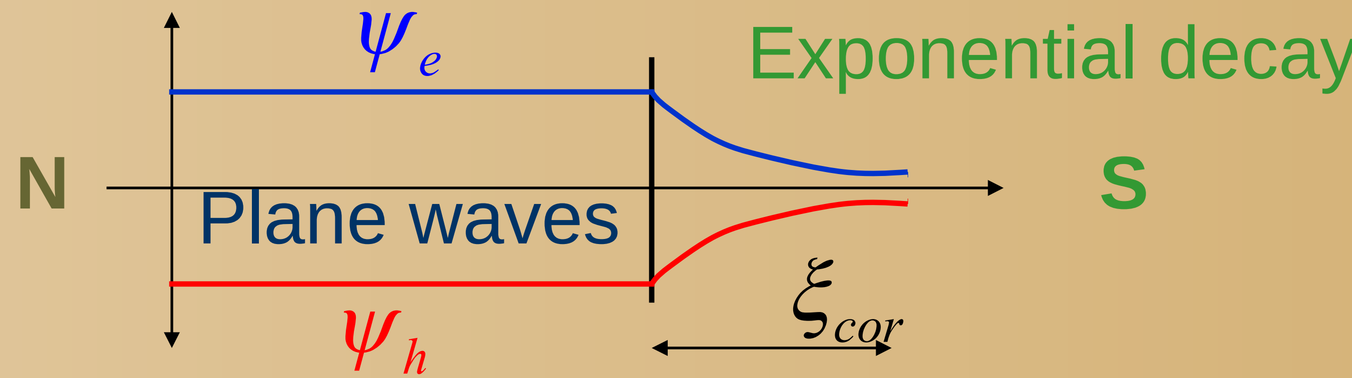
Energy: conserved

Charge: conserved! Since charge *2e* goes into the superconductor as a Cooper pair. (twee halen, een betalen)

Momentum: (almost) conserved! (while velocity flips)

Spin: conserved.

Andreev amplitude



For

$\boxed{E < \Delta}$ electron and hole states decay in the superconductor: Andreev reflection

Amplitude of the reflected hole wave:

sup. phase

Energy-dependent phase

$$\boxed{\psi_h = e^{-i\varphi - i\arccos \varepsilon / \Delta}\psi_e}$$

Remarkable universality: does not depend on the scattered wave

Scattering approach

Nanostructure: can be very complex

Can be modelled as:
waveguide with transport channels
+ potential barrier

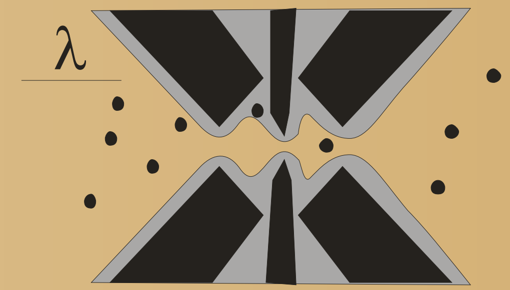
Essence: scattering matrix

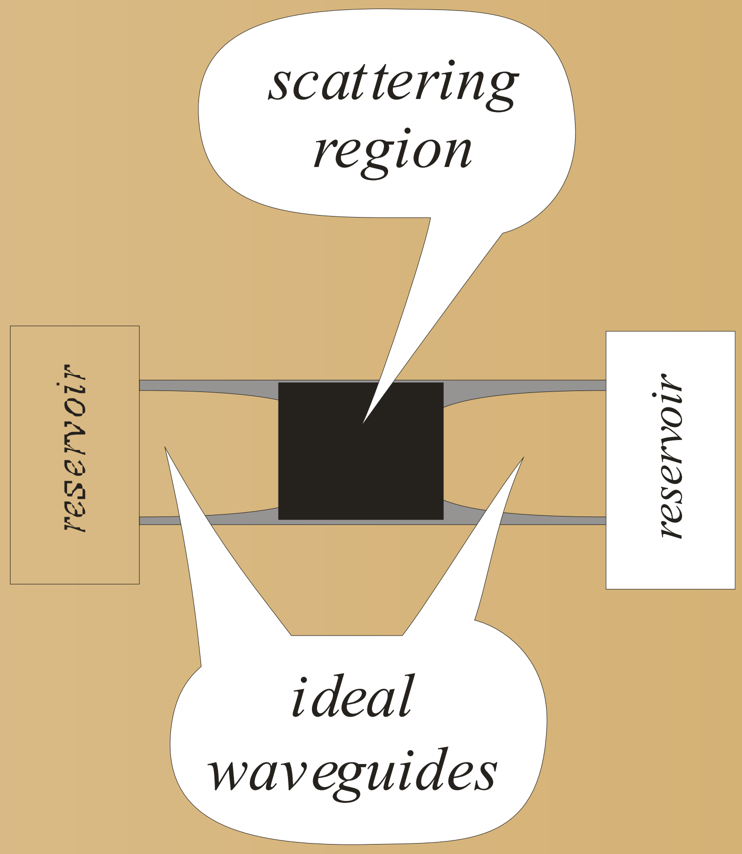
Incoming amplitudes $\vec{a}$

Outgoing amplitudes $\vec{b}$

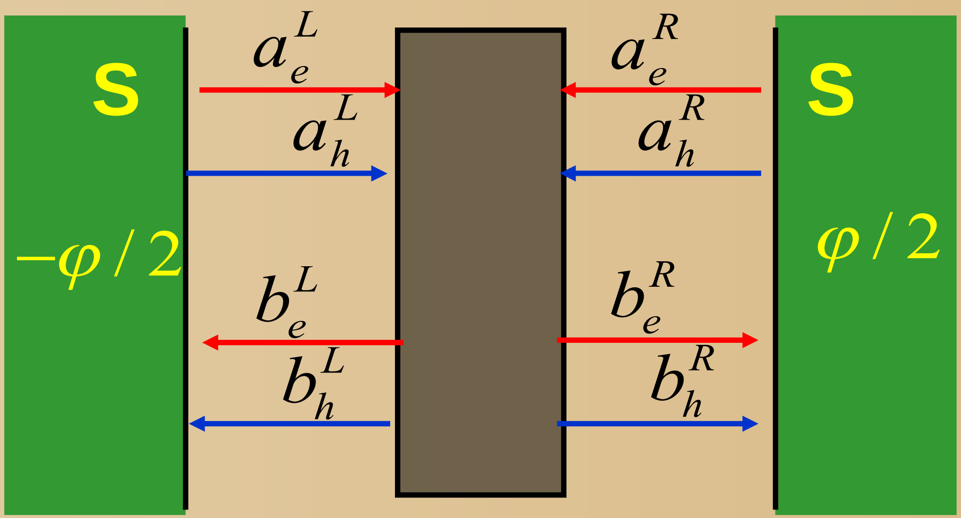
$$\vec{b} = \hat{s}\vec{a}$$

$$\begin{bmatrix} \vec{b}_L \\ \vec{b}_R \end{bmatrix} = \begin{bmatrix} \hat{r} & \hat{t}\,' \\ \hat{t} & \hat{r}\,' \end{bmatrix} \begin{bmatrix} \vec{a}_L \\ \vec{a}_R \end{bmatrix}$$





How and why to combine normal scattering and Andreev reflection



$$\begin{pmatrix} b_e^L \\ b_e^R \\ b_h^L \\ b_h^R \end{pmatrix} = \hat{s}_N \begin{pmatrix} a_e^L \\ a_e^R \\ a_h^L \\ a_h^R \end{pmatrix}, \hat{s}_N = \begin{pmatrix} \hat{s} & 0 \\ 0 & \hat{s}^* \end{pmatrix}$$

Scattering at the (short) nanostructure

$$\hat{s} = \begin{pmatrix} r & t \\ t & r' \end{pmatrix}$$

$$\begin{pmatrix} a_e^L \\ a_e^R \\ a_h^L \\ a_h^R \end{pmatrix} = \hat{s}_A \begin{pmatrix} b_e^L \\ b_e^R \\ b_h^L \\ b_h^R \end{pmatrix}, \hat{s}_A = \begin{pmatrix} 0 & 0 & e^{i\theta_{he}^L} & 0 \\ 0 & 0 & 0 & e^{i\theta_{he}^R} \\ e^{i\theta_{eh}^L} & 0 & 0 & 0 \\ 0 & e^{i\theta_{eh}^R} & 0 & 0 \end{pmatrix}$$

Beenakker formula

Therefore: $\boxed{\vec{\psi}_{out} = \hat{s}_N \vec{\psi}_{in} = \hat{s}_N \hat{s}_A \vec{\psi}_{out}}$

$\boxed{\det(\hat{s}_N \hat{s}_A - 1) = 0}$ Satistfied at certain θ = energy

$$\det\left(e^{i2\chi} - \hat{S}(\vec{\varphi}, E)\right) = 0$$

$$\hat{S}(\vec{\varphi}, E) = e^{i\hat{\varphi}} \hat{s}_{\mathrm{h}}(E) e^{-i\hat{\varphi}} \hat{s}_{\mathrm{e}}(E)$$

$$s_{\mathrm{h}}(E) = -\hat{g}\,\hat{s}_{\mathrm{e}}^{*}(-E)\hat{g}$$

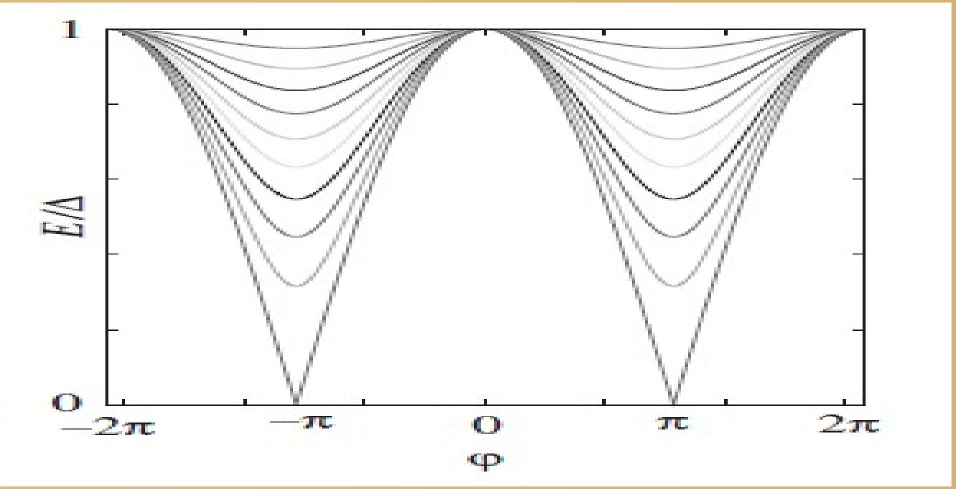
Energy dependence of scattering matrix

- At the scale of inverse dwell time
- Compared with Δ
- Short structure – can be disregarded
- Most of these lectures

Two-terminal (short) junction

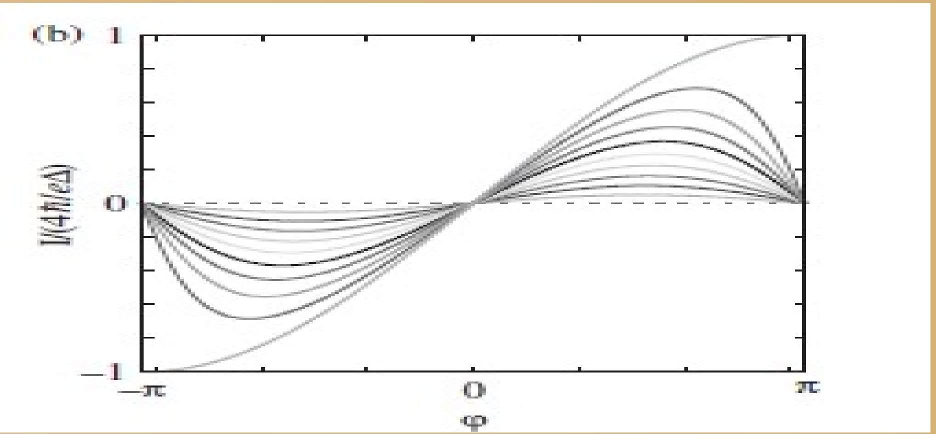
Many channels: bound state for each channel

$$E_p = \Delta\sqrt{1 - T_p \sin^2\left(\varphi/2\right)}$$



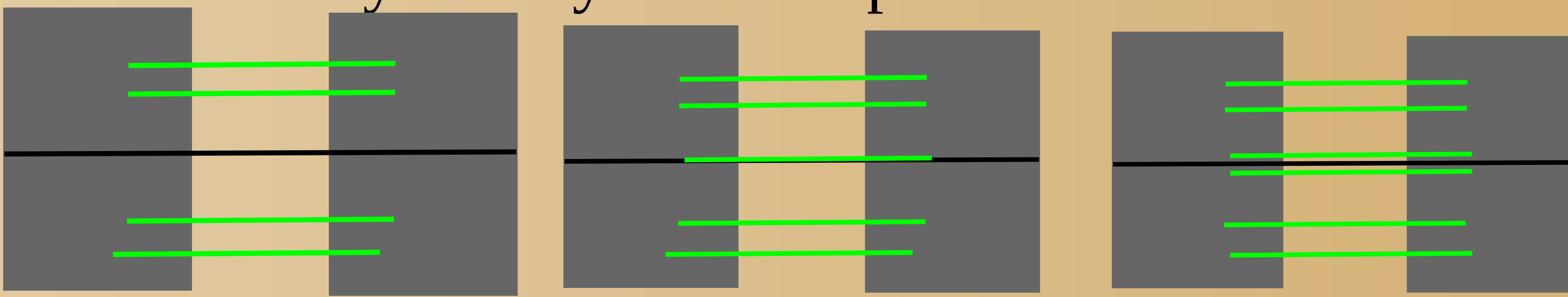
Phase-dependent part of ground state energy

$$E = -\sum_p E_p = -\Delta\sum_p \sqrt{1 - T_p \sin^2\left(\varphi/2\right)}$$



Majorana: idea

- Mirror symmetry of BdG spectrum

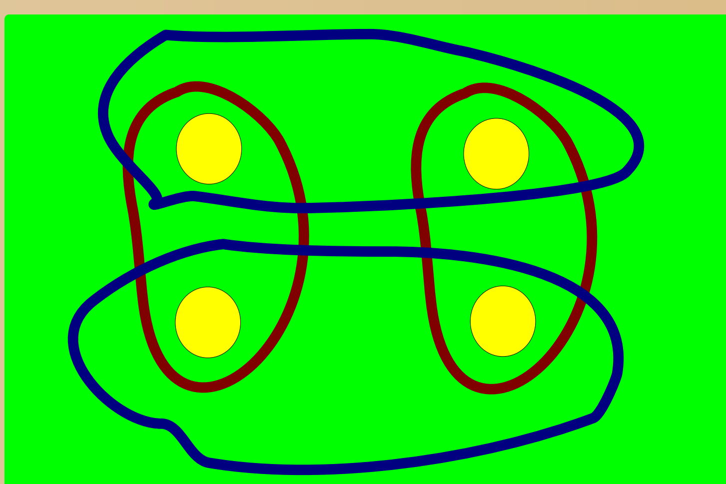


Usual, trivial

Topologically distinct = forbidden

Majorana pair: realistic

Majorana – localized entity – a placeholder. One quasiparticle – requires 2 placeholders



Highly degenerate state.

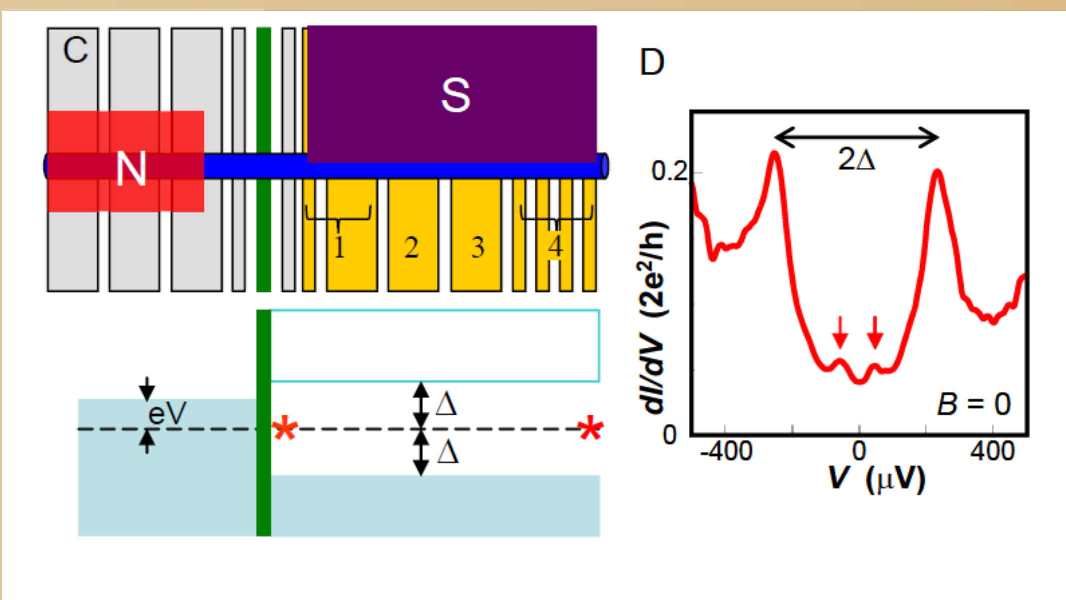
Interesting exchange statistics =>

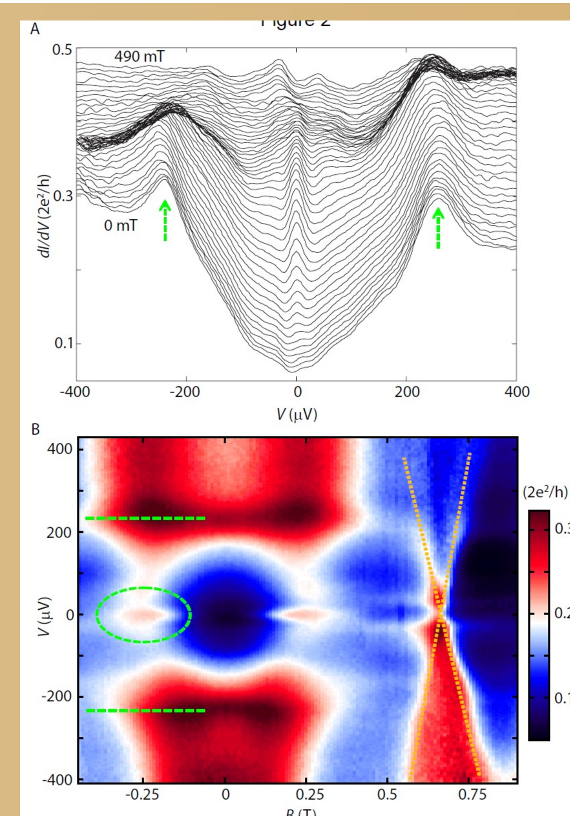
manipulation

Majorana: status: semiconducting nanowires

Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices

V. Mourik[1†], K. Zuo[1†], S.M. Frolov[1], S.R. Plissard[2], E.P.A.M. Bakkers[1,2], L.P. Kouwenhoven[1*]



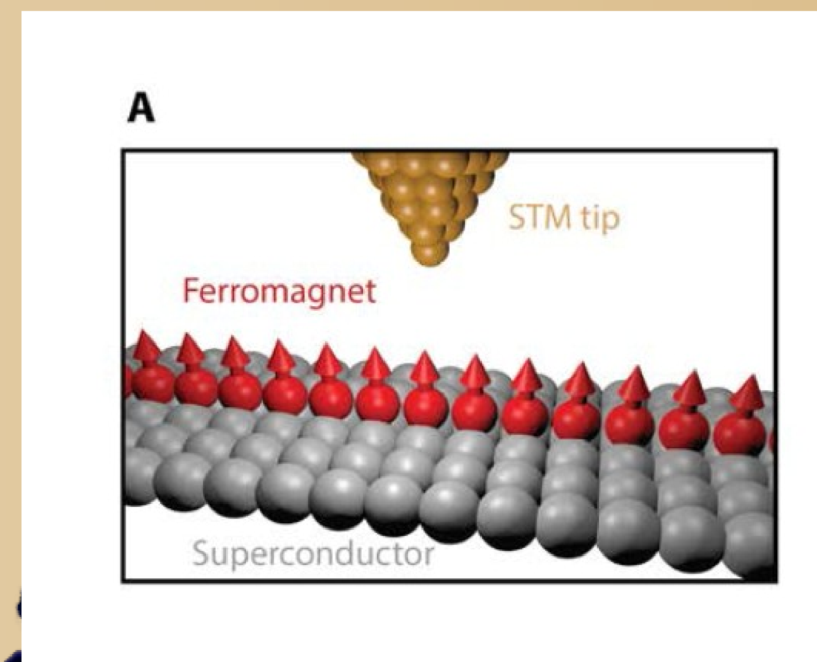


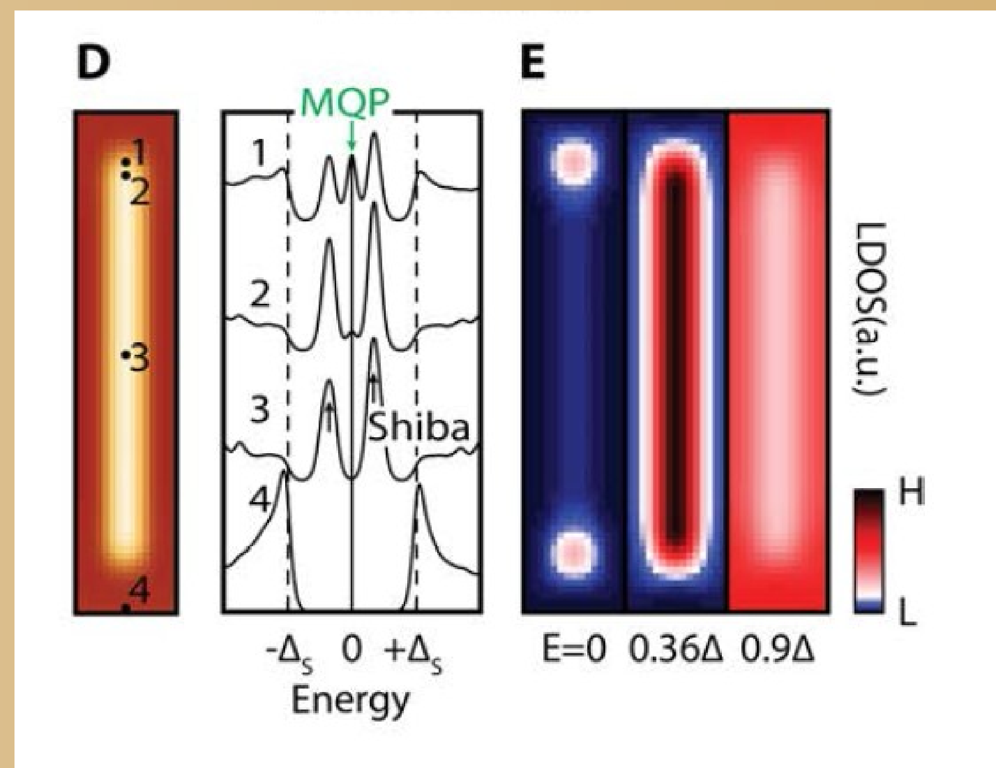
Long, strong spin-orbit, zeeman splitting

Majorana: status: magnetic chains

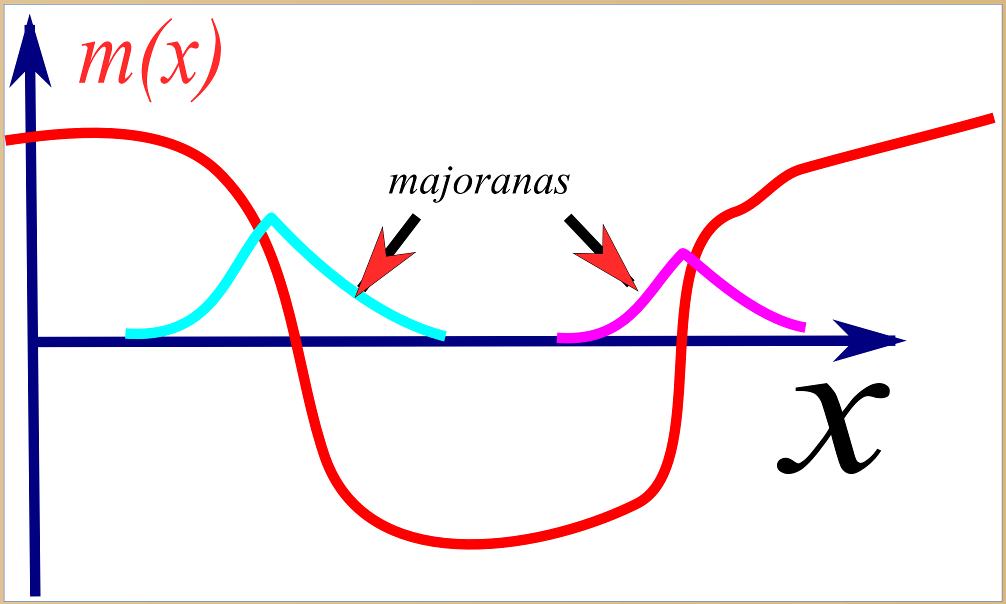
Observation of Majorana Fermions in Ferromagnetic Atomic Chains on a Superconductor

Stevan Nadj-Perge*[1], Ilya K. Drozdov*[1], Jian Li*[1], Hua Chen*[2], Sangjun Jeon[1], Jungpil Seo[1], Allan H. MacDonald[2], B. Andrei Bernevig[1] and Ali Yazdani[1]†



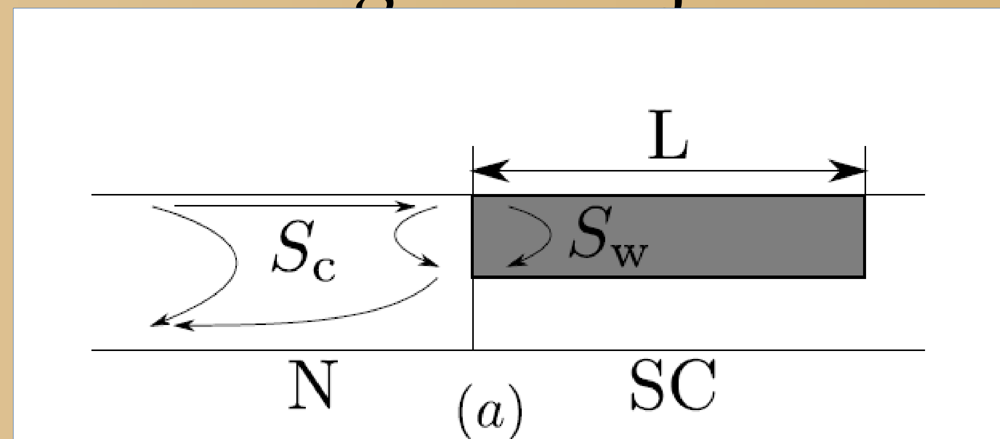


Majorana: simple example

- Minimum Hamiltonian: $\hat{H} = i\partial_x v\sigma_z + a(x)\sigma_y$
- Long-wave approximation near transition point
- Zero-energy: polarizations $\sigma_x = \pm 1$
- Localized states: at zeros of a(x)
- 

Majorana: scattering approach

- Combining normal scattering and majorana wire



Poles in energy dependence of the scattering matrix at small energy