Novel topologies in superconducting junctions

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Lecture #2

- Weyl points everywhere
- Berry curvature
- Multi-terminal superconducting junctions
- Weyl points in superconducting junctions
- Transconductance quantization
- Spin-orbit
- Semiclassical structures and Green functions
- Conservation laws in quantum mechanics
- Quantum circuit theory
- Quantum circuit theory for superconducting junctions
- Several simple examples

Weyl points everywhere

- L&L: levels do not cross along a line in a parameter space
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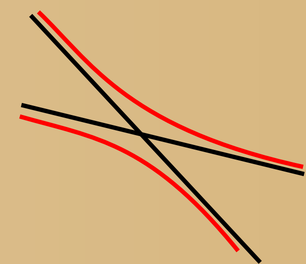


- Looks they can: one condition must be satisfied:

$$E_1(p) = E_2(p)$$

- Proof they do not: suppose they do at p=0 and look at 2x2 Hamiltonian

$$\hat{H} = const + \left[\begin{array}{cc} h_z & h_x + ih_y \\ h_x - ih_y & -h_z \end{array} \right]$$

- The crossing requires 3 conditions to fulfill:

$$h_z = h_x = h_y = 0$$

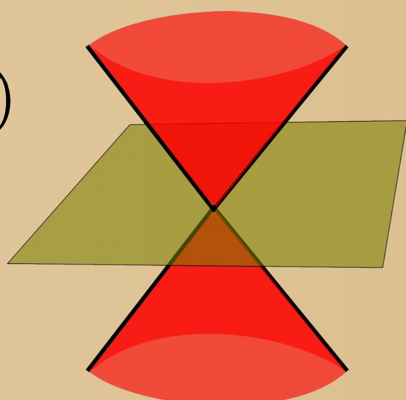
- Impossible!
  - Lazy Landau! Same reasoning: levels do cross in a 3d parameter space. In the vicinity of the Weyl point

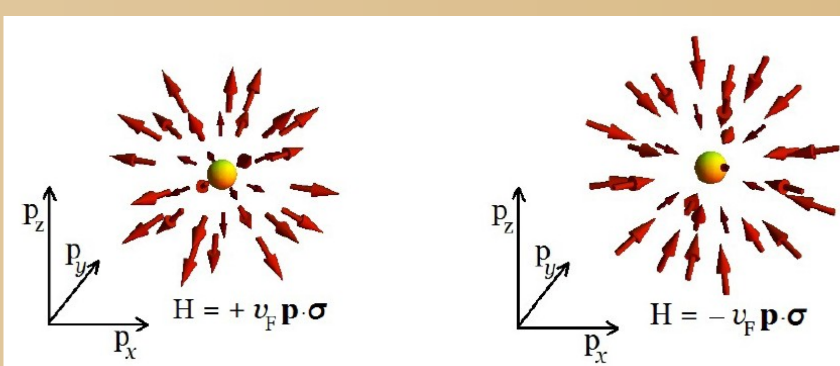
$$\hat{H} = \frac{\partial h_a}{\partial p_b} \sigma_a p_b$$

Weyl points in the particle spectrum and the bandstructure

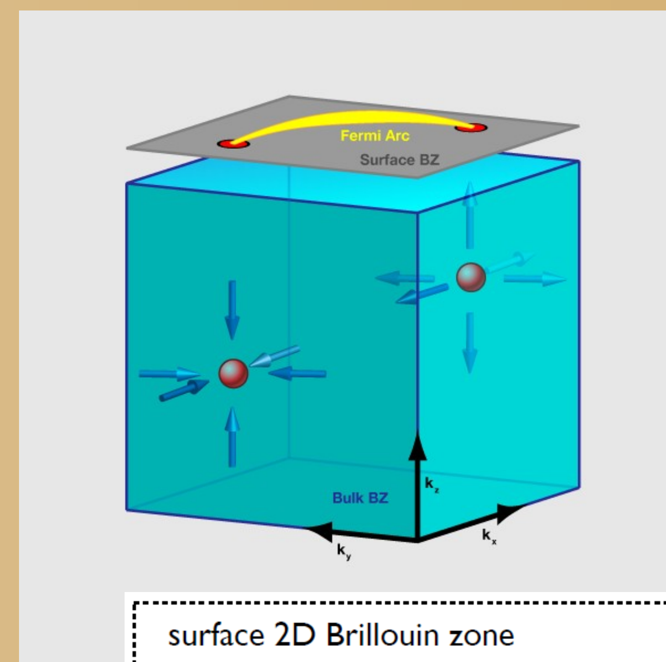
- Conical point in the spectrum. Massless relativistic fermions.

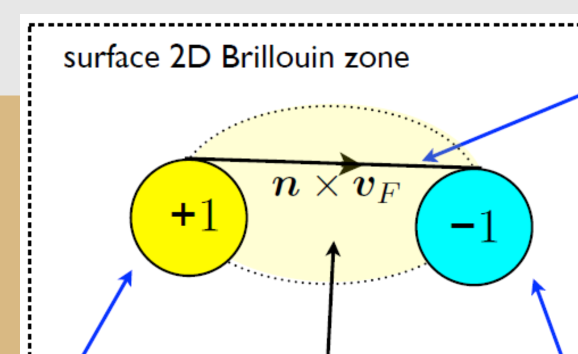
$$\hat{H} = \vec{\sigma} \cdot (\delta \vec{q})$$





- Weyl semimetal
- 2015 TaAs





Berry phase

- Adiabatic (no transitions) evolution of quantum state in the parameter space $$|\Psi_n(t)\rangle = e^{i\gamma_n(t)}\, e^{-\frac{i}{\hbar}\int_0^t dt'\,\varepsilon_n(\mathbf{R}(t'))}\,|n(\mathbf{R}(t))\rangle$$
- Berry phase $$\gamma_n(t) = i\int_0^t dt'\,\langle n(\mathbf{R}(t'))|\frac{d}{dt'}|n(\mathbf{R}(t'))\rangle = i\int_{\mathbf{R}(0)}^{\mathbf{R}(t)} d\mathbf{R}\,\langle n(\mathbf{R})|\nabla_{\mathbf{R}}|n(\mathbf{R})\rangle$$
-
- Berry connection: $$\mathcal{A}_\alpha = \langle n|\partial_\alpha|n\rangle$$
- Berry curvature: $$B_{\alpha\beta} = \partial_\beta\mathcal{A}_\alpha - \partial_\alpha\mathcal{A}_\beta$$
- Gauge-invariant quantity

Berry curvature

- Bandstructure: Eigenenergies *E(q_x,q_y,q_z)*
- From eigenstates: Berry curvature field
-
- 2D topological invariant(Chern number)
-

$$B_z(\vec{q}) = i\left\langle \frac{\partial\Psi}{\partial q_x} \middle| \frac{\partial\Psi}{\partial q_y} \right\rangle - \left\langle \frac{\partial\Psi}{\partial q_y} \middle| \frac{\partial\Psi}{\partial q_x} \right\rangle\Big)$$

- $C = \frac{1}{2\pi}\int dq_x dq_y B_z(\vec{q})$

- Electrostatic analogy
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- B.c. – el. field
- Weyl point – unit charge
- Chern number – el. flux
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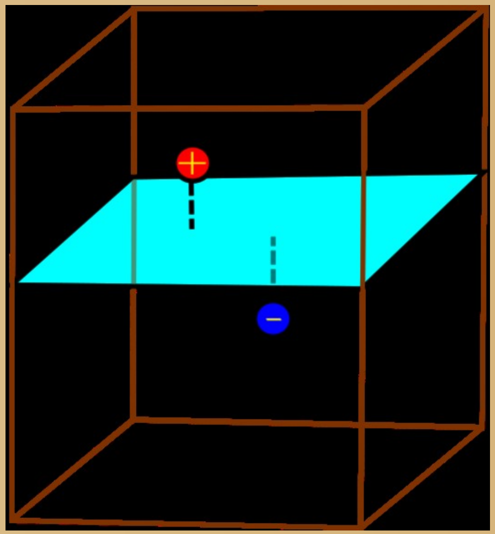
$$div\vec{B} = 0$$



Multi-terminal superconducting devices

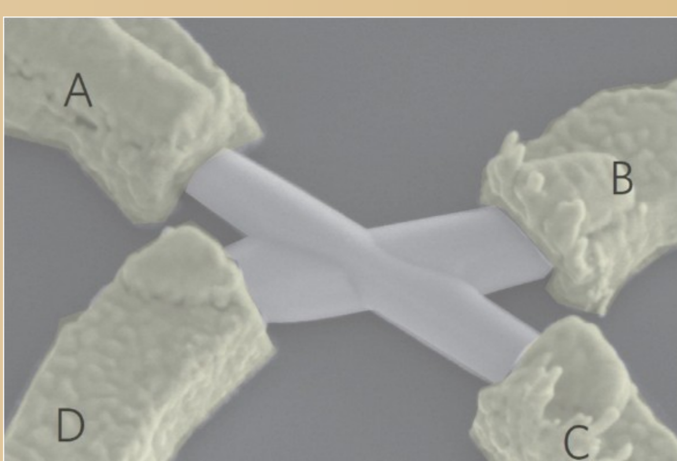
- Josephson junction
- More transparent – Andreev bound states
-
-
- More terminals – more superconducting phases
- same Andreev states

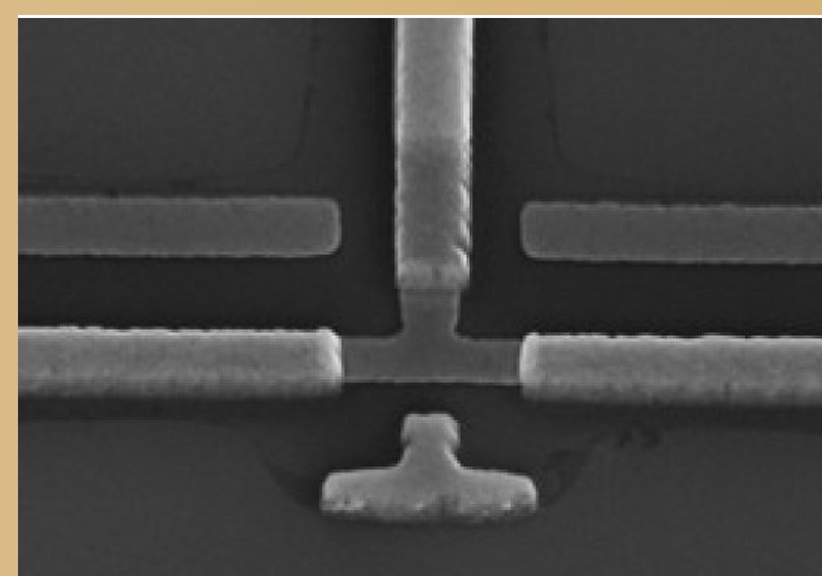
$$E = -E_J \cos\varphi$$

$$E = -\frac{1}{2}\sum_p E_p$$

$$E_p = \Delta\sqrt{1 - T_p \sin(\varphi/2)^2}$$

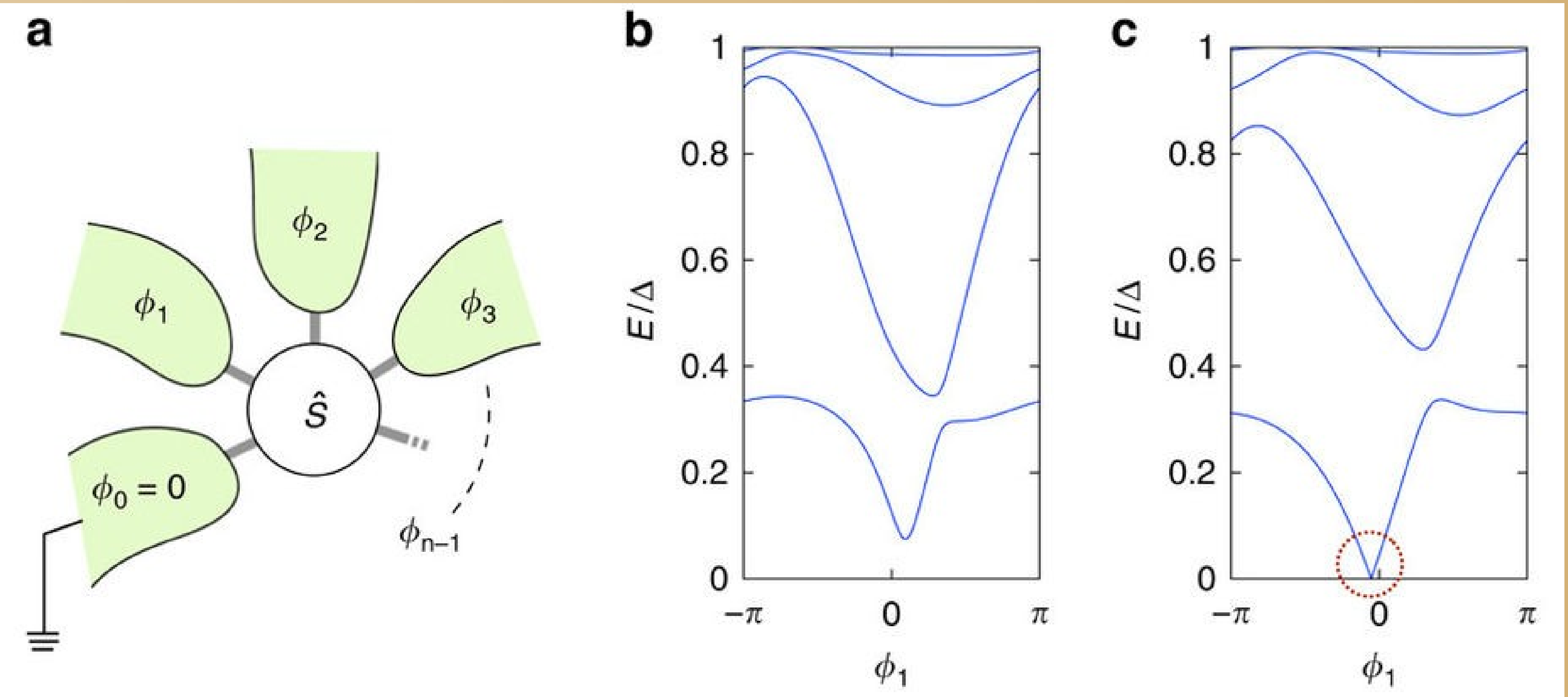
$$E_p(\phi_1, \cdots, \phi_{n-1})$$





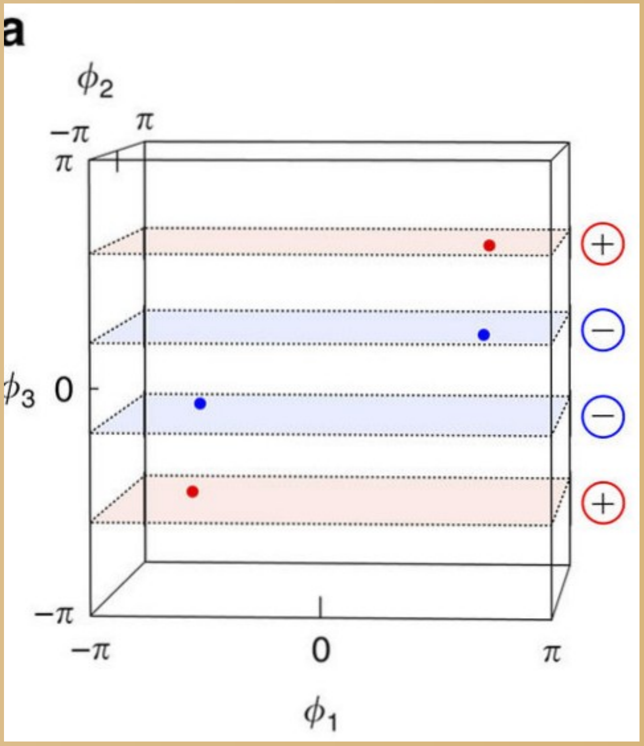
Weyl points in superconducting junctions

- Spectrum



Weyl points in superconducting junctions

- Specifics: crossing at zero energy
- Affects The Berry curvature of the many-body state
- $$B_{\alpha\beta} = -\frac{1}{2}\sum_k B_{\alpha\beta}^{(k)}$$
- Come in group of four



Berry curvature and transport

- Currents – functions of the phases
- Change the phases adiabatically
-
-
-
-
- Ser

$$I_\alpha(t) = \frac{2e}{\hbar}\frac{\partial E}{\partial \phi_\alpha} - 2e\dot{\phi}_\beta B^{\alpha\beta}$$

Leading order

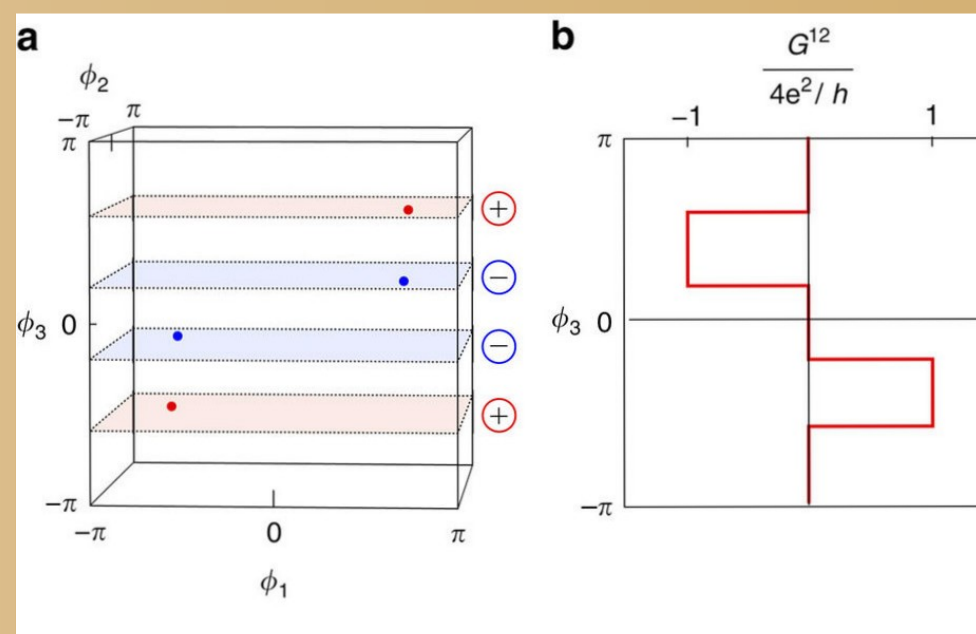
First correction

Transconductance quantization

- Apply (incommensurate) voltages to leads 1,2
- Keep the 3d lead at
- Phases are swept over BZ. Sup.current vanishes
- First correction remains ϕ_3
-
-
- Quantum Hall effect

$$I_1 = G_{12} V_2; \; I_2 = -G_{12} V_1$$

$$G_{12} \equiv (2e^2/\pi\hbar) C$$

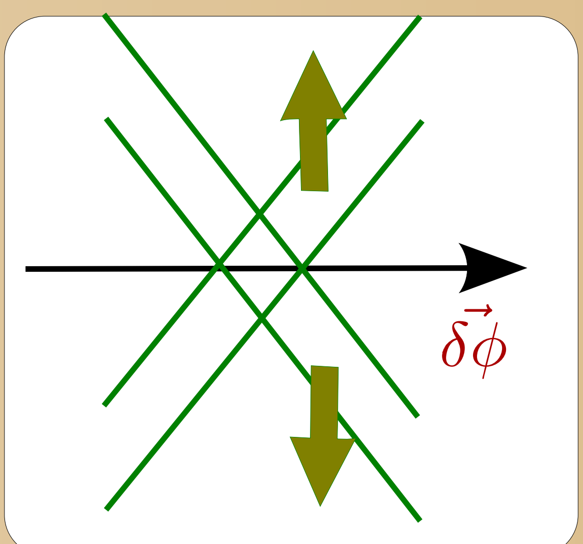


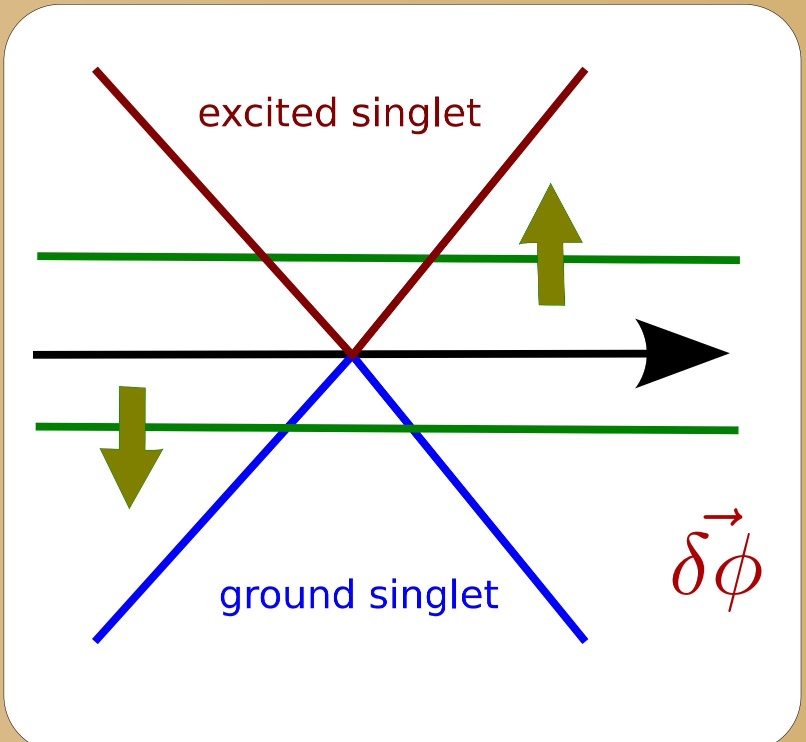
Spin-orbit

- This was without spin-orbit interaction :)
- Since there is no time reversibility,

$$\hat{H} = I\vec{\tau} \cdot \delta\vec{\phi} + \vec{\sigma} \cdot \vec{B}$$

- Spin-split cone or flat spinful states
- Weyl singularity departs(?) from zero energy





Semiclassical structures and Green functions

- Bigger than wavelength
- Big number of channels – big conductance
- Design – diffusive parts, tunnel junctions,
- Traditional semiclassical approach:
  - 2x2 (Nambu) matrix semiclassical Green function
- $\hat{G}(\vec{R}, \epsilon)$ $\hat{G}^2 = \hat{1}$

Conserving currents in quantum mechanics

$$E\psi = \hat{H}\psi = (-\frac{\hbar^2}{2m}\nabla^2 + U(\vec{r}))\psi(\vec{r})$$

Obeys Schr. equation

$$\vec{j}(\vec{r}) = \frac{-i\hbar}{2m}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*)$$

Let's try this expression

$$\vec{\nabla}\cdot\vec{j}(\vec{r}) = \frac{-i\hbar}{2m}\vec{\nabla}\cdot(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) =$$

$$\frac{-i\hbar}{2m}(\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi - \vec{\nabla}\psi\cdot\vec{\nabla}\psi^*) \Rightarrow 0$$

$$+\frac{-i\hbar}{2m}(\psi^*\vec{\nabla}^2\psi - \psi\vec{\nabla}^2\psi^*) \Rightarrow Sch.\ equation$$

$$\frac{i}{\hbar}(\psi^*(E-U(x))\psi - \psi(E-U(x))\psi^*) = 0$$

$$\vec{\nabla}\cdot\vec{j}(\vec{r}) = 0$$

More conserving currents in quantum mechanics

Wave function: mixture of red and blue

$$\bar{\psi} = \begin{bmatrix} \psi_r \\ \psi_b \end{bmatrix}$$

Time-reversable *H*:

Red and blue satisfy the same

$$\alpha, \beta = r, b$$

$$\vec{j}_{\alpha\beta}(\vec{r}) = \frac{-i\hbar}{2m}(\psi_\alpha^* \vec{\nabla}\psi_\beta - \psi_\beta \vec{\nabla}\psi_\alpha^*)$$

Magic current: 2x2 matrix

$$\vec{\nabla} \cdot \vec{j}_{\alpha\beta}(\vec{r}) = 0$$

Conserves if *H* is time-reversable

Quantum “currents” and “voltages”

Current: matrix
“Voltage”: matrix

Which matrix?
Depends on a problem

Proper
ty of
the
reserv
oir

$$\hat{G};\ \ Tr\hat{G} = 0, \quad \hat{G}^2 = \hat{1}$$

Eigenvalues:
±1

Examples:

$$\hat{G} = \begin{bmatrix} 1-2f & 2f \\ 2-2f & 2f-1 \end{bmatrix}$$

Filling factor

$$\hat{G} = \begin{bmatrix} 1-2\hat{\rho} & 2\hat{\rho} \\ 2-2\hat{\rho} & 2\hat{\rho}-1 \end{bmatrix}$$

Spin injection

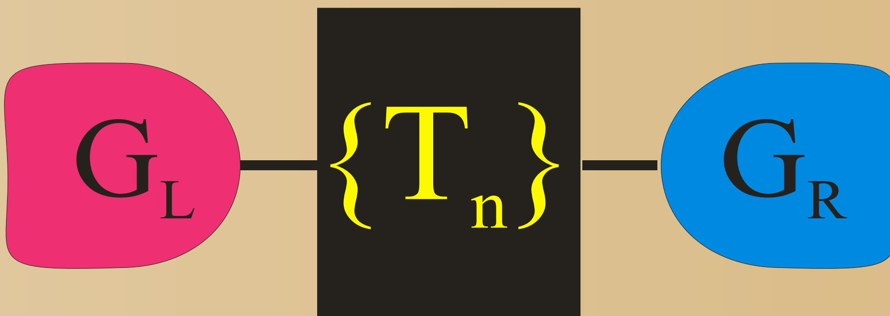
$$\hat{G} = \begin{bmatrix} 1-2f & 2f\exp(i\chi) \\ (2-2f)\exp(-i\chi) & 2f-1 \end{bmatrix}$$

Full counting
statistics

$$\hat{G} = \begin{bmatrix} g & -f^* \\ f & -g \end{bmatrix}$$

Superconductivit
y

Matrix current in a Landauer conductor

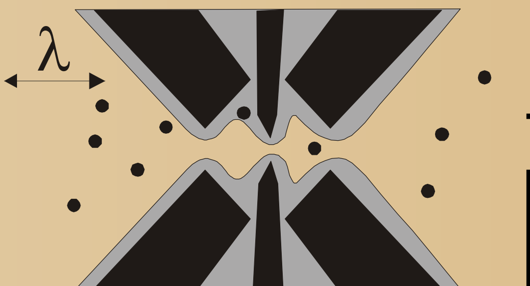


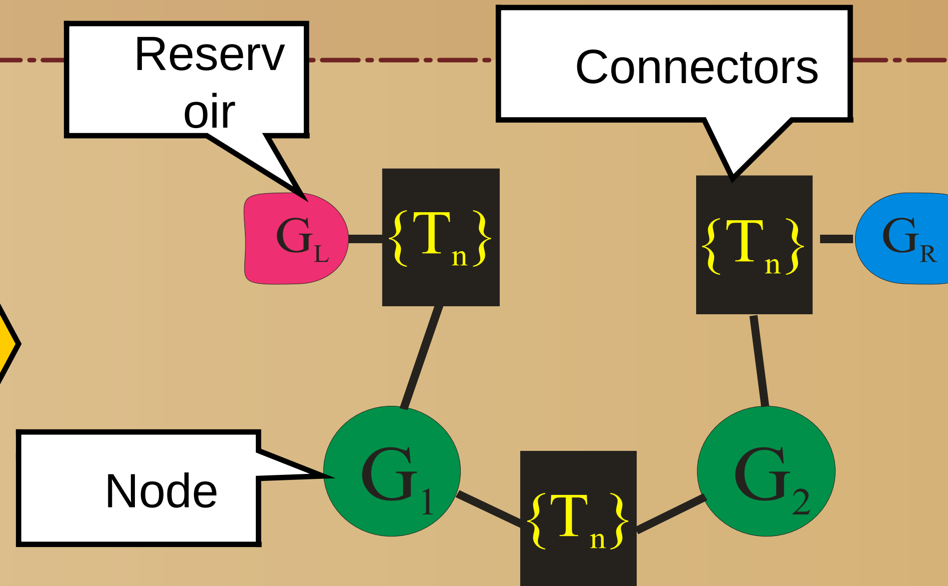
Transmissio
n
Eigenvalue
s

$$\hat{I} = \frac{e^2}{\pi\hbar}\sum_n \frac{T_n(\hat{G}_L\hat{G}_R - \hat{G}_R\hat{G}_L)}{2 + \frac{T_n}{2}(\hat{G}_L\hat{G}_R + \hat{G}_R\hat{G}_L - 2)}$$

Matrix “voltage”
on the left(right)
end

Template of a quantum circuit theory





Assumption :
node = reservoir
requires :
conductance $\gg G_Q$

G_R, G_L –fixed

$\hat{I} = \hat{I}(\hat{G}_1, \hat{G}_2)$
-given

G_1, G_2-
unknown

How to find
G_1, G_2, I?

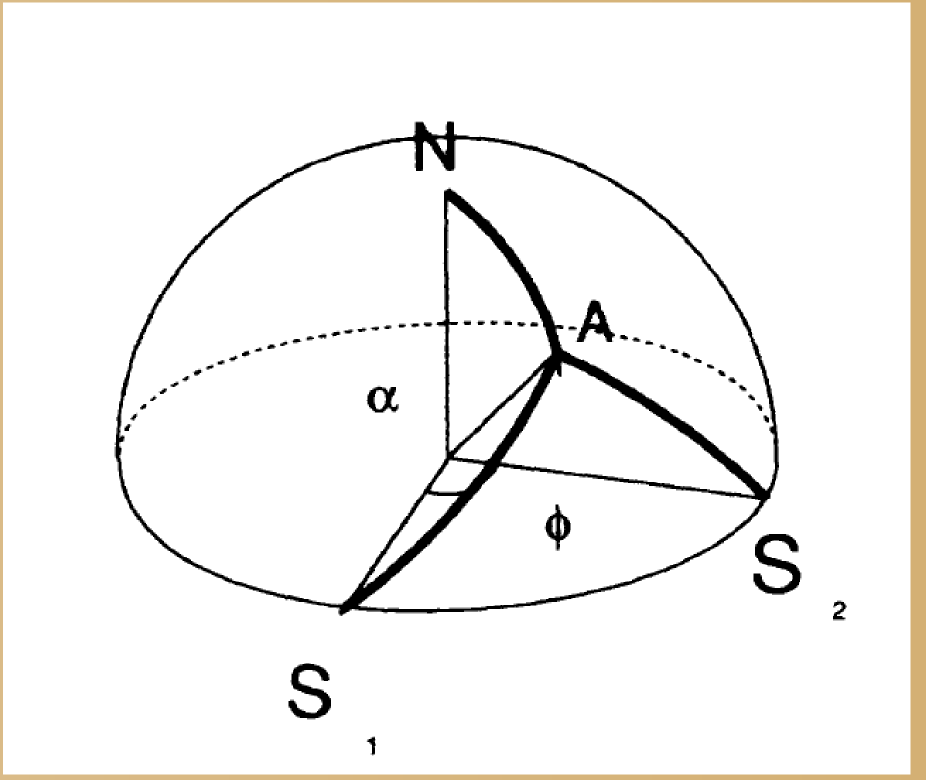
$$\textit{for each node} \sum_{connectors} \hat{I} = 0$$

(A) Quantum circuit theory for superconducting junctions

- Imaginary energy
- **G** – a real 3d vector on a hemisphere
  - Normal metal – North pole
  - Sup. Terminal at zero energy

    – at the equator
  - **G** – in nodes
  - Connectors – rubber threads.
  - ~~Leakage terminals~~
  - Their “elastic energy”

$$S(\mathbf{G}_1 \cdot \mathbf{G}_2)$$



Conductor types

α – angle between G’s

- General $S = \frac{1}{2}\sum_p \ln(1 - T_p \sin^2(\alpha/2))$
- Diffusive $S = \frac{G_D}{8}\alpha^2$
- Tunnel $S = -\frac{G_T}{2}\sin^2\frac{\alpha}{2}$
- Ballistic $S = -G_B \ln\cos\frac{\alpha}{2}$