Dissipative magnetic dynamics

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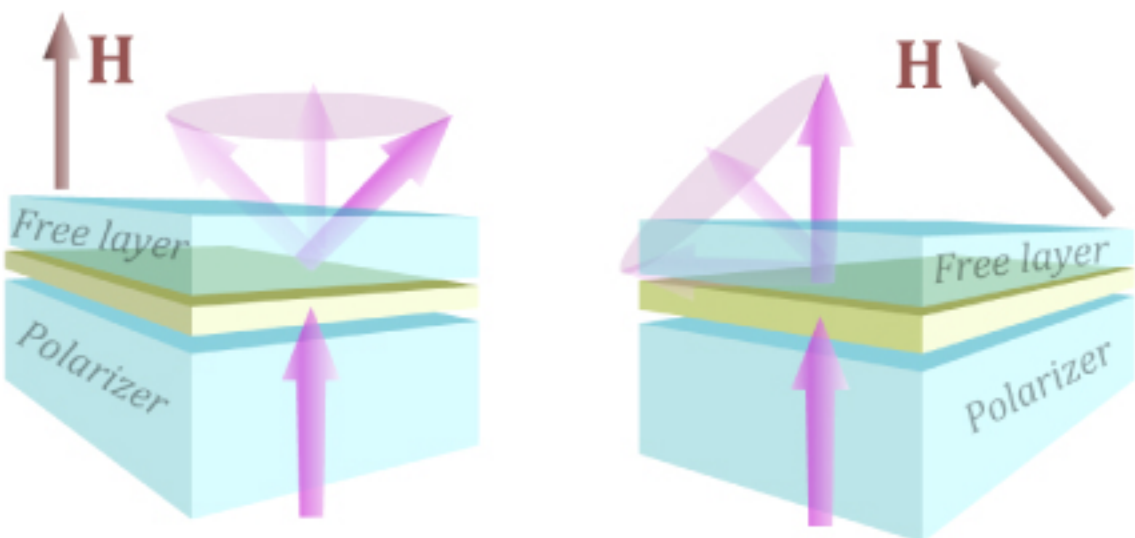
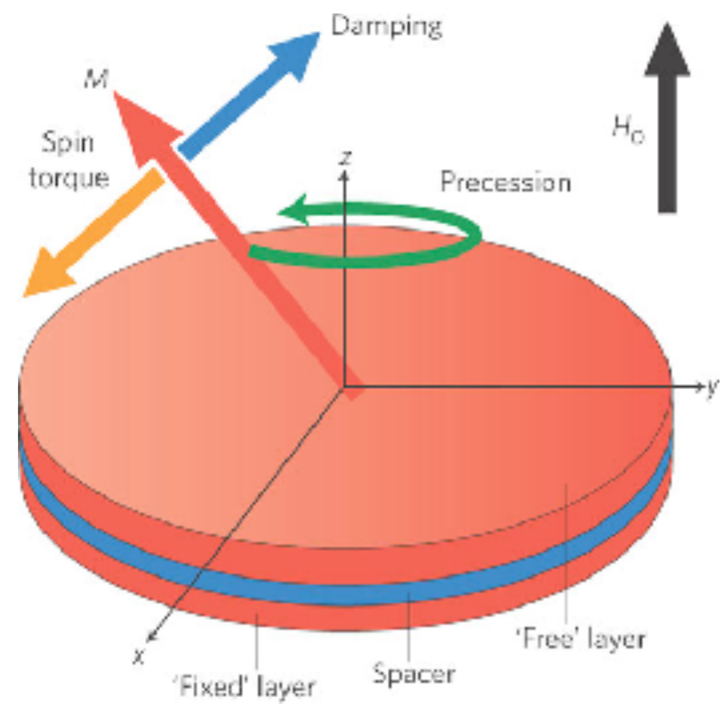
Lecture 1

Plan

1. Motivation: magnetic tunnel junctions
2. Motivation: mesoscopic Stoner instability
3. Theoretical method: path integral (functional bosonization)
   a) U(1) - Coulomb blockade
   b) SU(2) - mesoscopic Stoner
4. Ambegaokar-Eckern-Schön (AES) effective action
   a) U(1)
   b) SU(2) - LLG-Langevin equations

Spin-Torque-Oscillators





$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \mathbf{M}\times\mathbf{B} - \alpha\frac{\mathbf{M}}{M_0}\times\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} + \mathbf{M}\times\left(\frac{\mathbf{I}_s}{S}\times\frac{\mathbf{M}}{M_0}\right)$$

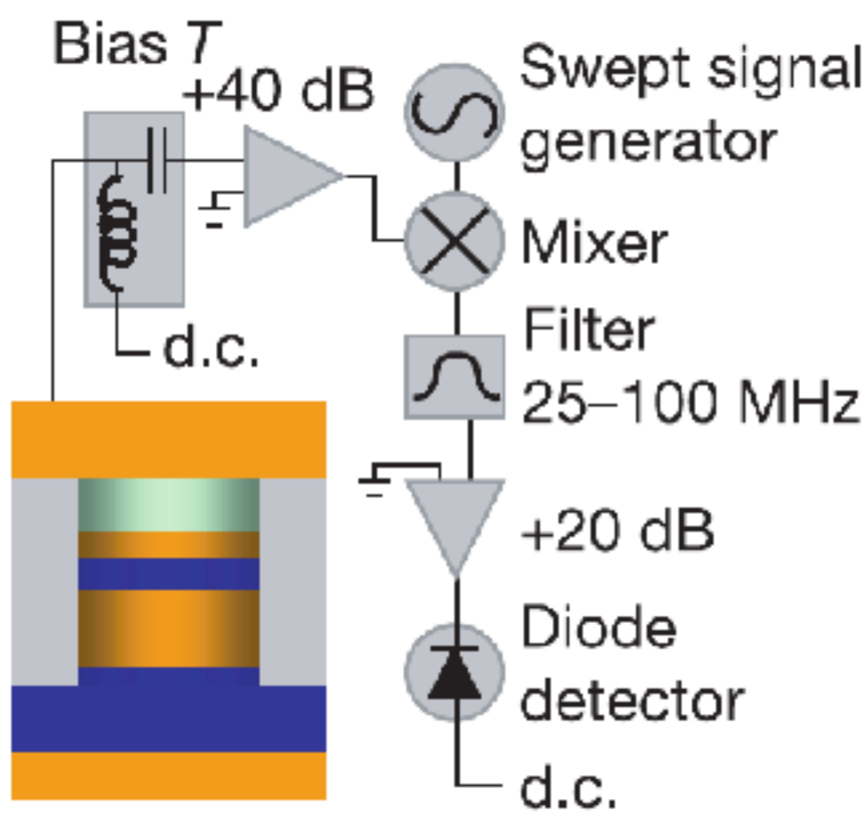
Landau & Lifshitz, Phys. Z. Sowietunion 8, 153 (1935)

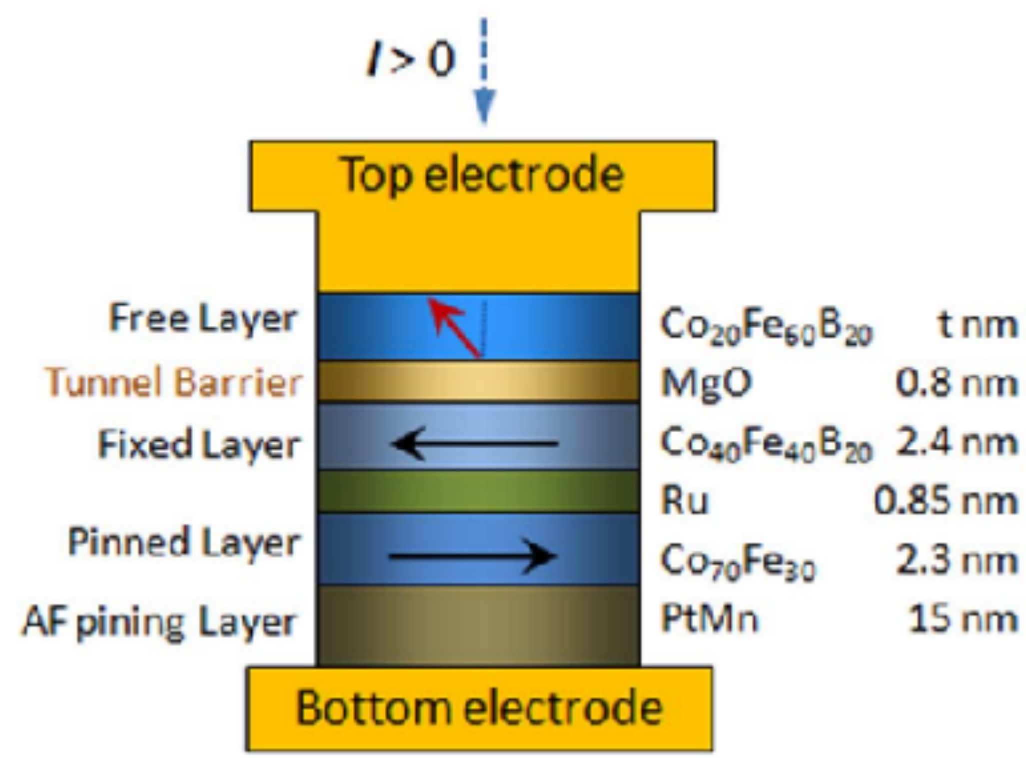
T.L. Gilbert (1955, 2004)

L. Berger, Phys. Rev. B 54, 9353 (1996)

J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996)

Spin-Torque-Oscillators



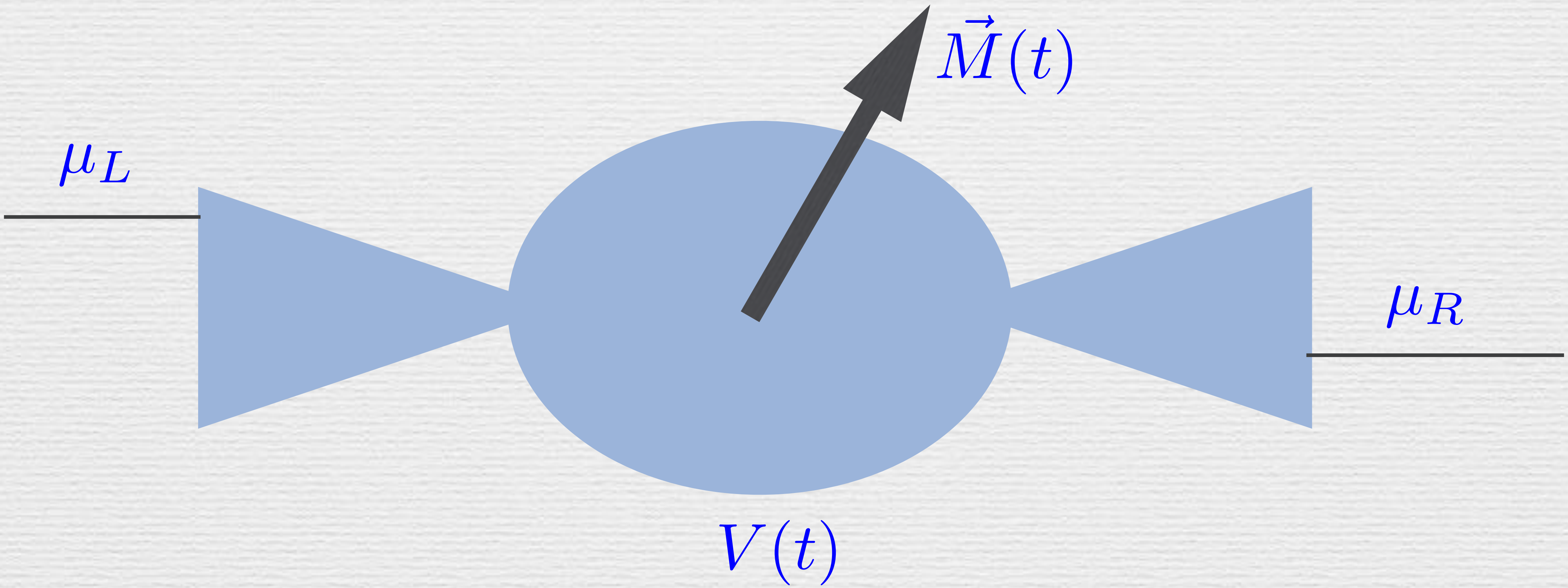


S. Kiselev et al.
Nature 425, 380 (2003)

Z. Zeng et al.
Scientific Reports 3, 1426 EP (2014)

Goal

Proper description in terms of slow collective variables:
magnetization $\vec{M}(t)$ and electric potential $V(t)$



Universal Hamiltonian

A. V. Andreev, A. Kamenev, Phys. Rev. Lett. 81, 3199 (1998)

I. L. Kurland, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 62, 14886 (2000)

$$H = H_0 + H_C + H_J + H_\lambda$$

$$H_0 = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma}$$

Coulomb $\quad H_C = E_C \left(\hat{N} - N_0 \right)^2 \qquad \hat{N} = \sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma}$

Exchange $\quad H_J = -J\hat{\mathbf{S}}^2 \qquad \hat{\mathbf{S}} = \sum_{\alpha} a^\dagger_{\alpha,\sigma_1} \mathbf{S}_{\sigma_1\sigma_2} a_{\alpha,\sigma_2}$

Cooper $\quad H_\lambda = \lambda T^\dagger T \qquad T = \sum_{\alpha} a_{\alpha,\uparrow} a_{\alpha,\downarrow}$

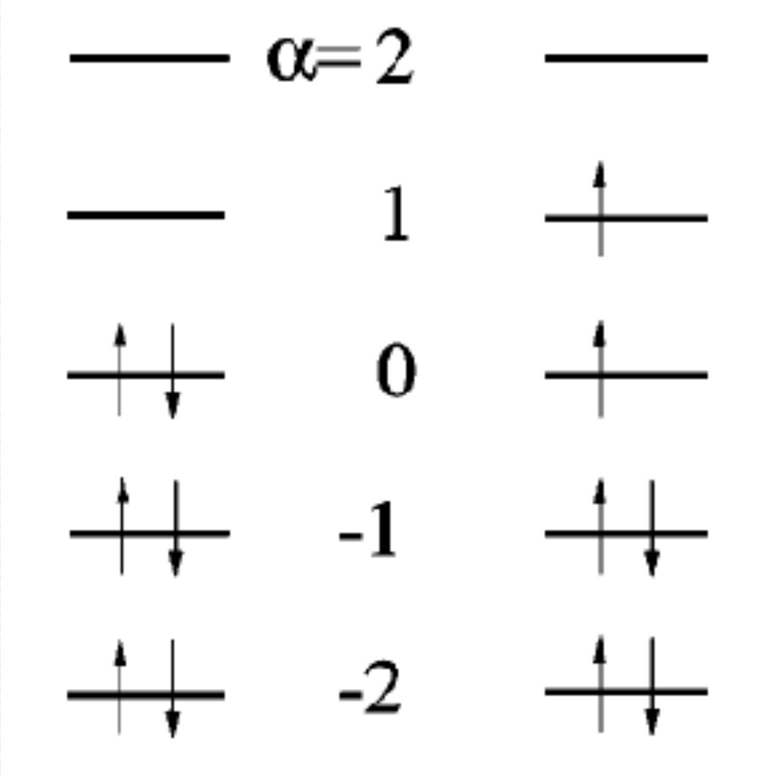
Mesoscopic Stoner Instability

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

$$\langle \hat{\mathbf{S}}^2 \rangle = S(S+1)$$

Balance of Energy

$$+\delta - 2J$$



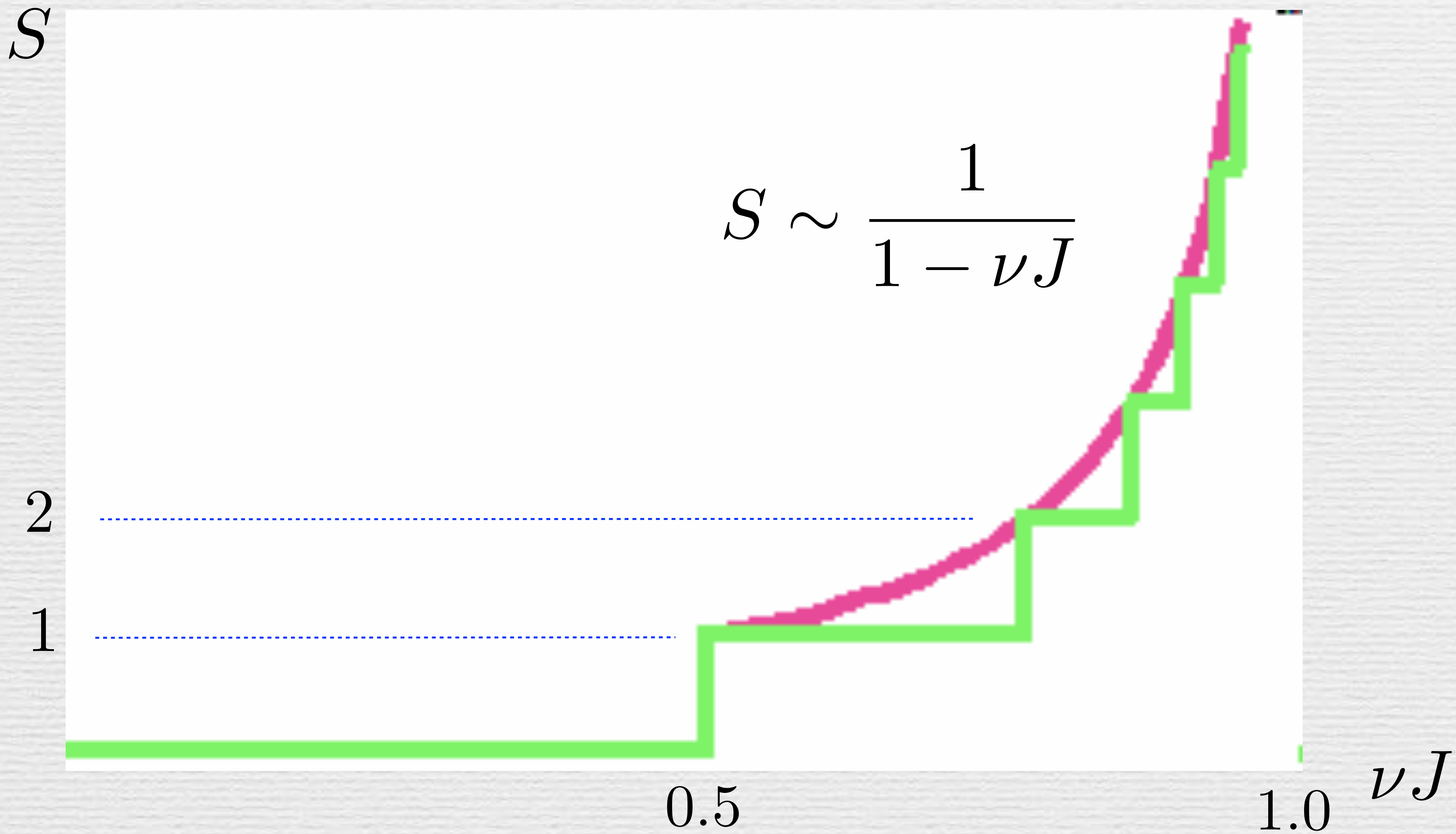
I. L. Kurland, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 62, 14886 (2000)

Mesoscopic Stoner Instability

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

$$\nu = \frac{1}{\delta}$$

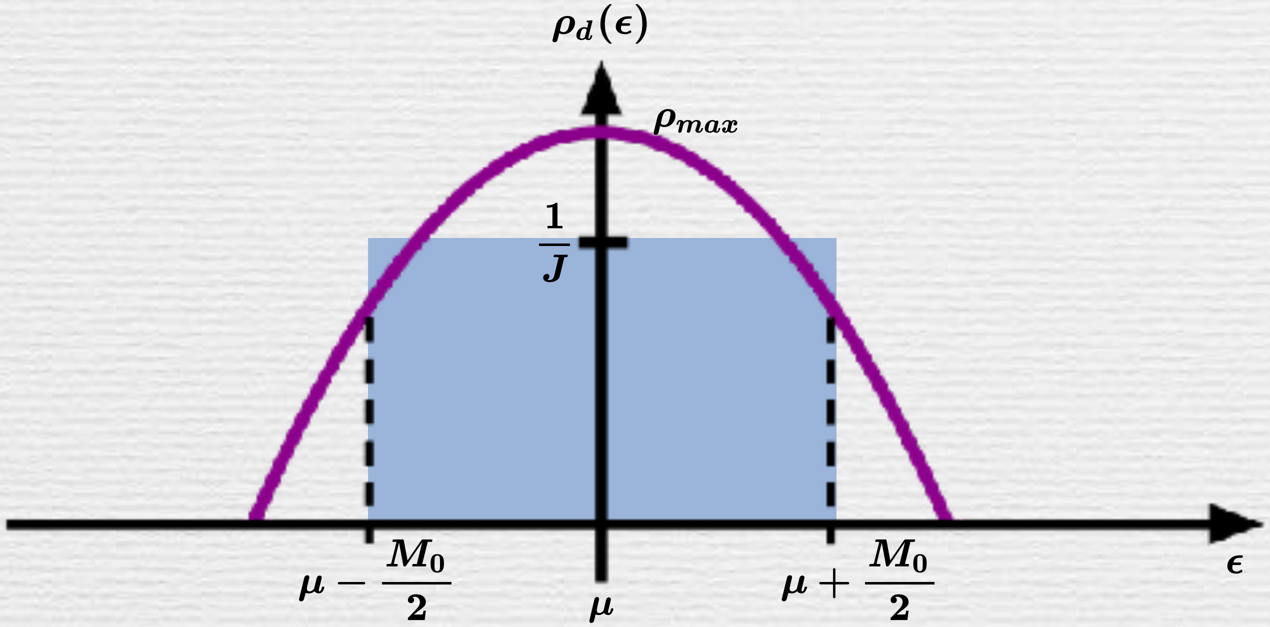
denisty of states

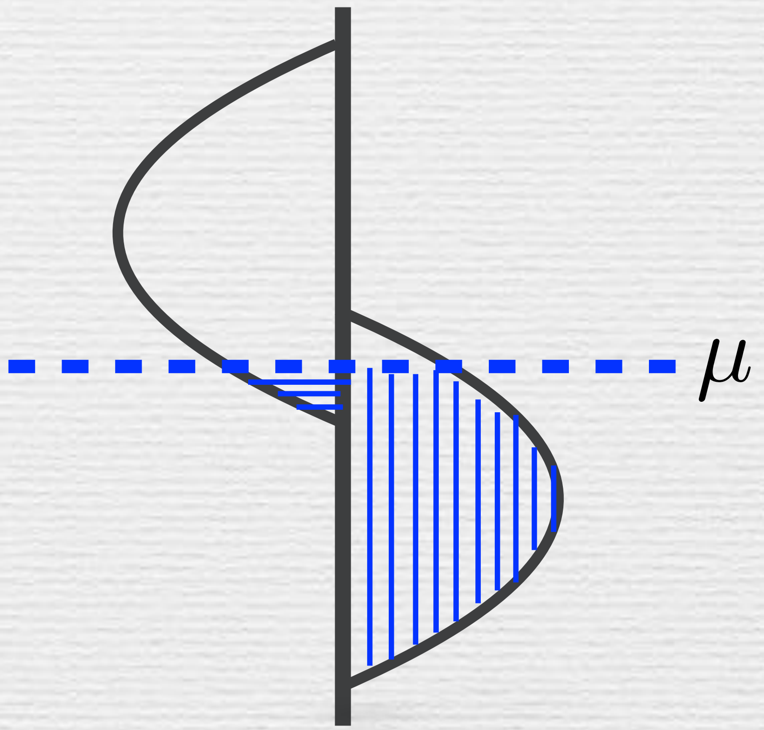


Stoner Ferromagnet

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

denisty of states





$$\frac{1}{M_0} \int_{\mu - M_0/2}^{\mu + M_0/2} d\epsilon\, \rho(\epsilon) = \frac{1}{J}$$

Theoretical Method

(functional bosonization)

Abelian case: Coulomb part

A. Kamenev, Y. Gefen, Phys. Rev. B 54, 5428 (1996)

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} + E_C(\hat{N} - N_0)^2 \qquad \mathcal{Z} = \int D\bar{\Psi} D\Psi \, e^{\mathcal{S}}$$

Hubbard-Stratonovich $\hat{N} = \sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma}$

$$\mathcal{S}_{\Psi,V} = \int_0^\beta d\tau \left[\sum_\alpha \bar{\Psi}_\alpha \left[-\partial_\tau - \epsilon_\alpha + \mu - iV(\tau) \right] \Psi_\alpha - \frac{V^2(\tau)}{4E_C} + iV(\tau) N_0 \right]$$

$$\mathcal{S}_V = \sum_\alpha \mathrm{tr} \ln \left(-\partial_\tau - \epsilon_\alpha + \mu - iV \right) - \int_0^\beta d\tau \left[\frac{V^2}{4E_C} - iV N_0 \right]$$

Abelian case: Coulomb part

A. Kamenev, Y. Gefen, Phys. Rev. B 54, 5428 (1996)

$$\mathcal{S}_\alpha = \mathrm{tr}\,\ln\left[-\partial_\tau - \epsilon_\alpha + \mu - iV\right]$$

$$\mathcal{S}_\alpha = \mathrm{tr}\,\ln\left[R^{-1}\left\{-\partial_\tau - \epsilon_\alpha + \mu - iV\right\}R\right]$$

Periodic boundary cond.

$$\Psi(\tau) \to R(\tau)\Psi(\tau)$$

$$R(\tau+\beta) = R(\tau)$$

$R \in U(1)$

$R = e^{-i\phi}$

$$\mathcal{S}_\alpha = \mathrm{tr}\,\ln\left[-\partial_\tau - \epsilon_\alpha + \mu - iV - R^{-1}\partial_\tau R\right]$$

$$iV(\tau) + R^{-1}\partial_\tau R = i(V - \dot{\phi}) \to iV_0$$

Zero mode

Winding numbers

$$-\pi/\beta < V_0 < \pi/\beta$$

$$\mathcal{Z}(\mu) \propto \int dV_0\, Z_{\mathrm{free}}(\mu - iV_0)\, e^{-\frac{\beta V_0^2}{4E_C} + i\beta V_0 N_0}$$

Non-Abelian case

(functional bosonization)

Non-Abelian case: Exchange part

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - J\hat{\mathbf{S}}^2 \qquad \hat{\mathbf{S}} = \sum_\alpha a^\dagger_{\alpha,\sigma_1} \mathbf{S}_{\sigma_1\sigma_2} a_{\alpha,\sigma_2}$$

$$\mathcal{S}_{\Psi,\mathbf{M}} = \int_0^\beta d\tau \left[\sum_\alpha \bar{\Psi}_\alpha \left(-\partial_\tau - \epsilon_\alpha + \mu - \mathbf{M}\cdot\mathbf{S}\right)\Psi_\alpha - \frac{|\mathbf{M}|^2}{4J} \right]$$

$$\mathcal{S}_{\mathbf{M}} = \sum_\alpha \mathrm{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - \mathbf{M}\cdot\mathbf{S}\right) - \int_0^\beta d\tau \frac{|\mathbf{M}|^2}{4J}$$

Non-Abelian

M.N.Kiselev,Y.Gefen, Phys. Rev. Lett. 96, 066805 (2006)

I.Burmistrov,Y.Gefen,M.Kiselev,,Pis'ma v ZhETF 92, 202 (2010)

Exact solution

I.Burmistrov,Y.Gefen,M.Kiselev, Pis'ma v ZhETF 92, 202 (2010)

$$\mathcal{S}_{\mathbf{M}} = \sum_{\alpha} \mathrm{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - \mathbf{M}\cdot\mathbf{S}\right) - \int_0^\beta d\tau \frac{|\mathbf{M}|^{\mathbf{2}}}{4J}$$

Non-Abelian

Wei-Norman-Kolokolov substitution

$$T\exp\left[\int d\tau\,\vec{M}(\tau)\cdot\vec{\sigma}\right] = \prod_n \exp\left[\int d\tau\,A_n(\tau)\sigma_n\right]$$

Jacobian $\mathbf{M} \leftrightarrow A_n$ Path integral $\mathcal{Z} = \int D\mathbf{M}\, e^{S_\Phi}$

$$\chi = \frac{1}{3}\frac{\partial \ln \mathcal{Z}}{\partial J} = \frac{1}{2}\frac{\nu}{(1-J\nu)} + \frac{\beta}{12}\left[\frac{1}{(1-\nu J)^2} - 1\right]$$

Pauli Curie

Non-Abelian case

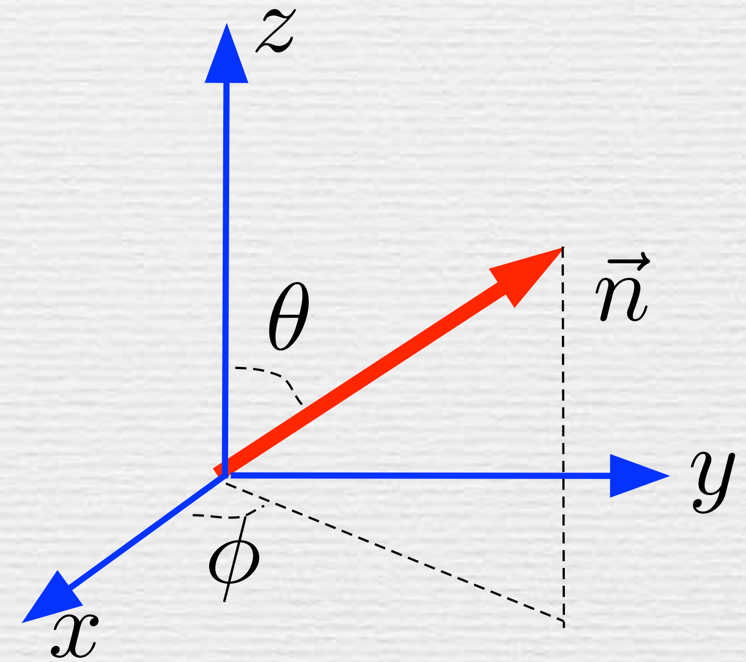
adiabatic approach

Geometric adiabatic solution

A. Saha et al. Annals of Phys., 327 (10), 2543 (2012)

$$\mathcal{S}_{\mathrm{M}} = \sum_{\alpha} \mathrm{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - M(\tau)\vec{n}(\tau)\cdot\vec{\mathbf{S}}\right) - \int_0^\beta d\tau\, \frac{M^2(\tau)}{4J}$$

Transform to rotating frame



$$\vec{n}\cdot\vec{\mathbf{S}} = R\,S_z\,R^\dagger$$

$$R \in SU(2)/U(1)$$

$$\mathcal{S}_{\Phi} = \sum_{\alpha} \mathrm{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - M(\tau)\,S_z - R^{-1}\partial_\tau R\right) + \ldots$$

Non-Abelian vector potential
i.a., Berry phase

Rotation: convenient representation

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[-\frac{i\psi}{2}\sigma_z\right]$$

$$R \in SU(2)/U(1)$$

$$\Psi(\tau) \to R(\tau)\Psi(\tau)$$

$$R(\tau+\beta) = R(\tau)$$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\chi}{2}\sigma_z\right]$$

periodic

$$\psi = -\phi + \chi$$

$$\chi(\tau+\beta) = \chi(\tau) + 4\pi n$$

Adiabatic expansion, 0-th order

$\mathbf{M}(\tau)$ large and slow

$$\mathcal{S}_\Phi = \sum_\alpha \operatorname{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - M(\tau)\,\frac{\sigma_z}{2} - R^{-1}\partial_\tau R\right) - \int_0^\beta d\tau\,\frac{M^2(\tau)}{4J}$$

$$-\beta\Omega(\Phi_0) = \sum_\alpha \operatorname{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - M_0\,\frac{\sigma_z}{2}\right) - \frac{\beta M_0^2}{4J}$$

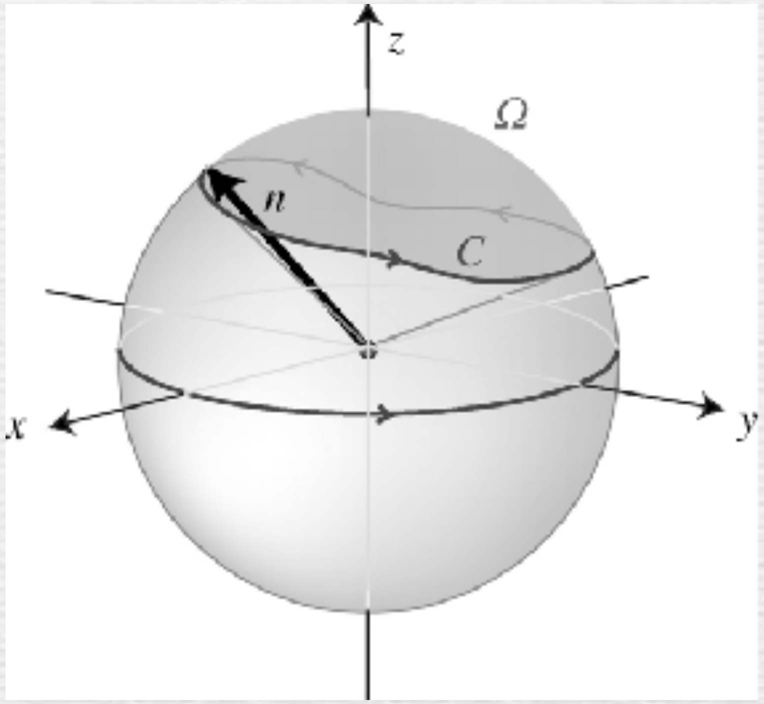
$$\Omega = const. + \left(\frac{1}{J} - \nu\right)\frac{M_0^2}{4} = const. + \frac{M_0^2}{4J_*}$$

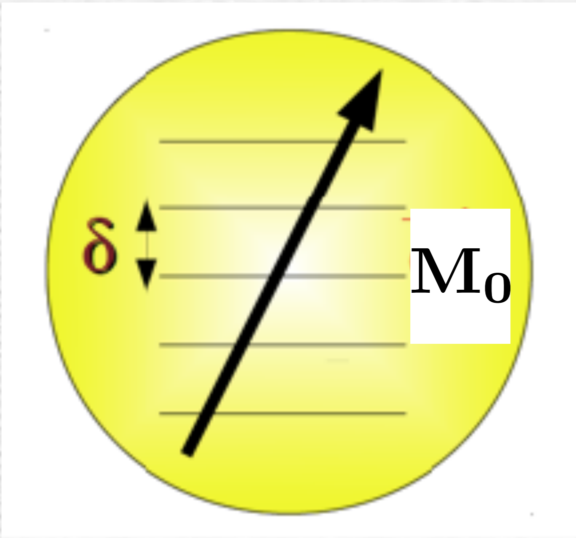
Stoner instability $\quad J_* = \dfrac{J}{1-\nu J} \to \infty$

Adibatic expansion, 1-st order

$$\mathcal{S}_{\mathbf{M}} = \sum_{\alpha} \mathrm{tr}\ln\left(-\partial_\tau - \epsilon_\alpha + \mu - M(\tau)\, S_z - R^{-1}\partial_\tau R\right) + \dots$$

$$\mathcal{S}_M \approx -\beta\Omega(M_0) + iS\int_0^\beta d\tau\,(1-\cos\theta)\dot{\phi}$$





$$S \approx \frac{\bar{\rho}_{dot} M_0}{2} \gg 1$$

Total spin

Berry's phase
WZNW action

Integrating over Berry's phase

$$\mathcal{S}_M \approx -\beta\Omega(M_0) + iS\int_0^\beta d\tau\,(1-\cos\theta)\dot{\phi}$$

Close to Stoner

$$\int \mathcal{D}\vec{n}\,\exp\left[iS\int_0^\beta d\tau\,\dot{\phi}\,(1-\cos\theta)\right]$$

$$\approx \int \mathcal{D}\phi\mathcal{D}y\,\exp\left[iS\int_0^\beta d\tau\,\dot{\phi}y\right]$$

$$\approx \prod_{m=1}^{N}\left(\frac{1}{2\beta S\omega_m}\right)^2$$

$S \gg 1$

$y \equiv 1-\cos\theta \ll 1$

Mainly "small" contours contribute

Adiabatic condition

$\omega_m \leq \Phi_0$

Proper Jacobian crucial

Integrating over Berry's phase

$$\mathcal{S}_M \approx -\beta\Omega(M_0) + iS\int_0^\beta d\tau\,(1-\cos\theta)\dot{\phi}$$

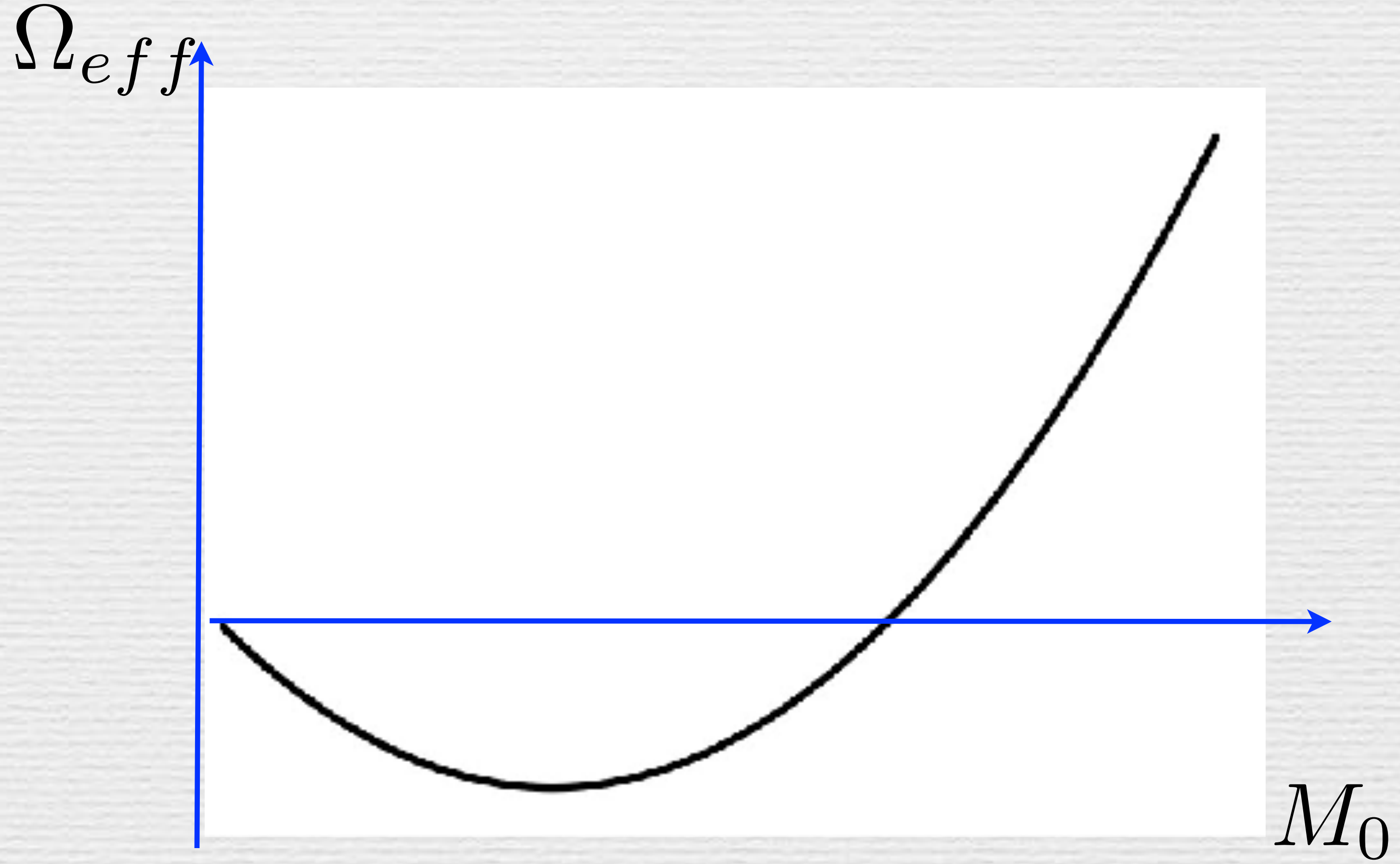
$$\mathcal{Z} \propto \int_0^\infty dM_0 4\pi M_0^2 \exp\left[-\frac{\beta M_0^2}{4J^*}\right]\cdot\frac{\sinh\left[\frac{\beta M_0}{2}\right]}{\frac{\beta M_0}{2}} \propto \left(\frac{J^*}{J}\right)^{3/2}\exp\left[\frac{\beta J^*}{4}\right]$$

Effective Potential

$$\mathcal{S}_M \approx -\beta\Omega(M_0) + iS\int_0^\beta d\tau\,(1-\cos\theta)\dot{\phi}$$

$$\mathcal{Z} \propto \int_0^\infty dM_0 4\pi M_0^2 \exp\left[-\frac{\beta M_0^2}{4J^*}\right]\cdot\frac{\sinh\left[\frac{\beta M_0}{2}\right]}{\frac{\beta M_0}{2}}$$

$$\Omega_{eff} \sim \frac{M_0^2}{4J^*} - \frac{|M_0|}{2}$$



L. Kurland, I. L. Aleiner, and B. L. Altshuler, Phys. Rev. B 62, 14886 (2000)

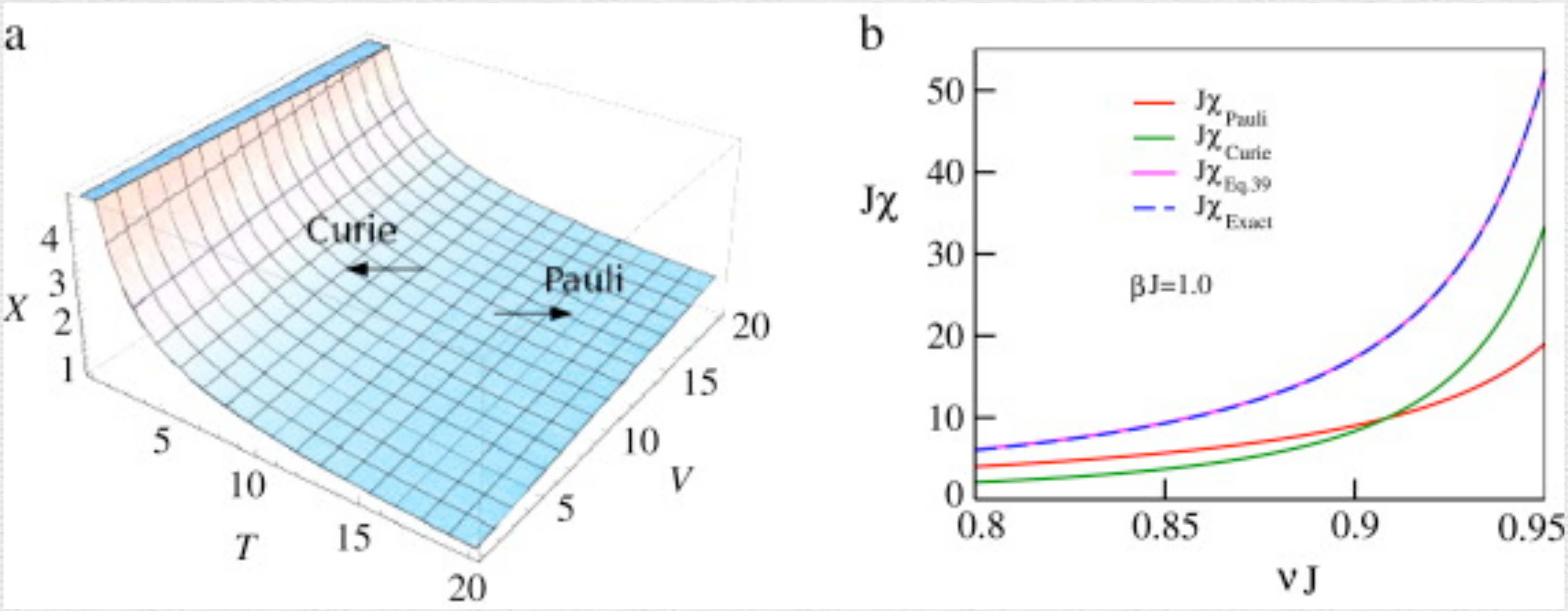
I. S. Burmistrov, Y. Gefen, M. N. Kiselev, Phys. Rev. B 85, 155311 (2012)

Result: susceptibility

$$\chi = \frac{1}{3}\frac{\partial \ln \mathcal{Z}}{\partial J} = \frac{1}{2}\frac{\nu}{(1-J\nu)} + \frac{\beta}{12}\frac{1}{(1-\nu J)^2}$$

Pauli Curie

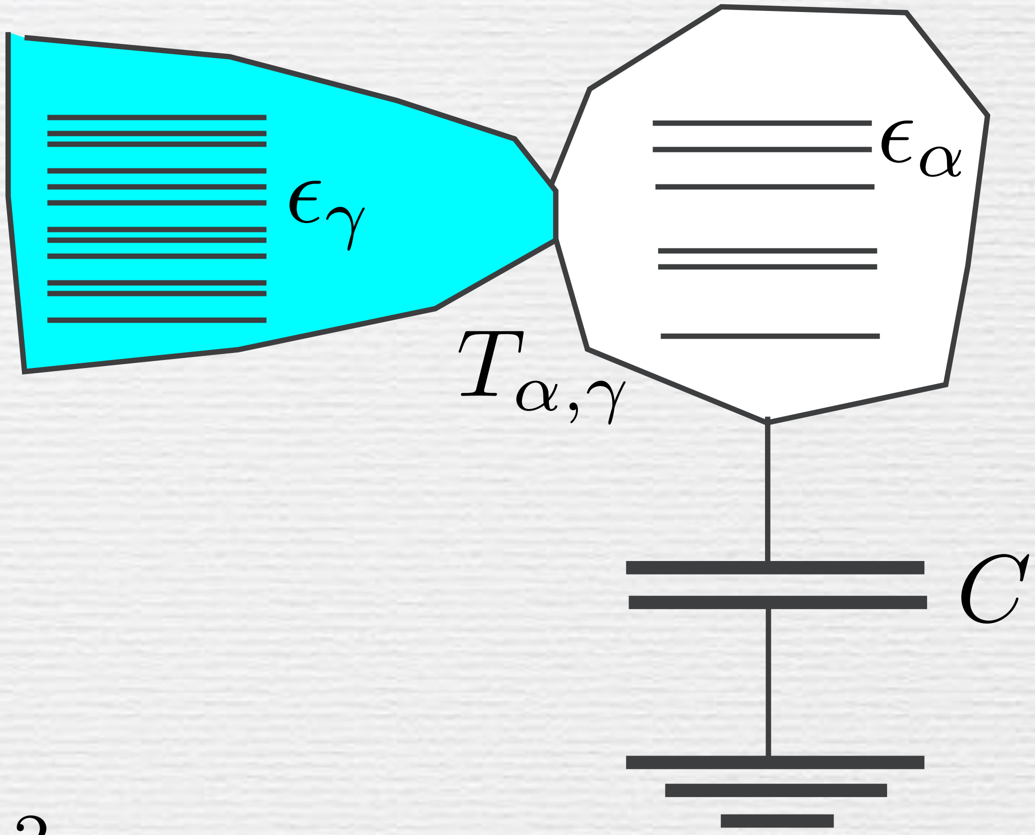
I.Burmistrov,Y.Gefen,M.Kiselev, Pis'ma v ZhETF 92, 202 (2010)



AES action: Abelian U(1) case

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. **48**, 1745-1748 (1982)

U(1) case



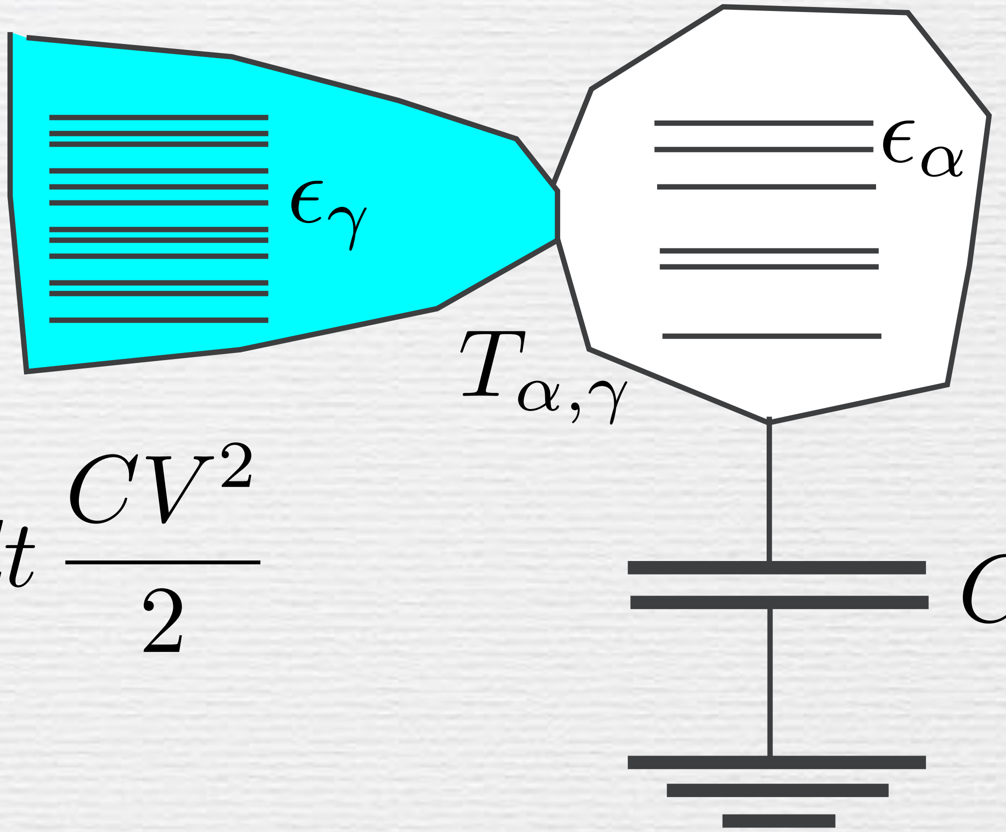
$$H = H_{dot} + H_{lead} + H_t$$

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha \psi^\dagger_{\alpha,\sigma}\psi_{\alpha,\sigma} + E_C(\hat{N} - N_0)^2$$

$$H_{lead} = \sum_{\gamma,\sigma} \epsilon_{\gamma,\sigma} c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma}$$

$$H_T = \sum_{\alpha,\gamma,\sigma} T_{\alpha,\gamma} \psi^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + h.c.$$

U(1) case



$$i\mathcal{S}_V = \mathrm{tr}\ln\begin{pmatrix} G_{dot}^{-1} & -T \\ -T^\dagger & G_{lead}^{-1}\end{pmatrix} + i\int dt\,\frac{CV^2}{2}$$

$$G_{dot}^{-1} = i\partial_t - \epsilon_\alpha - eV(t)$$

$$G_{lead}^{-1} = i\partial_t - \epsilon_\gamma$$

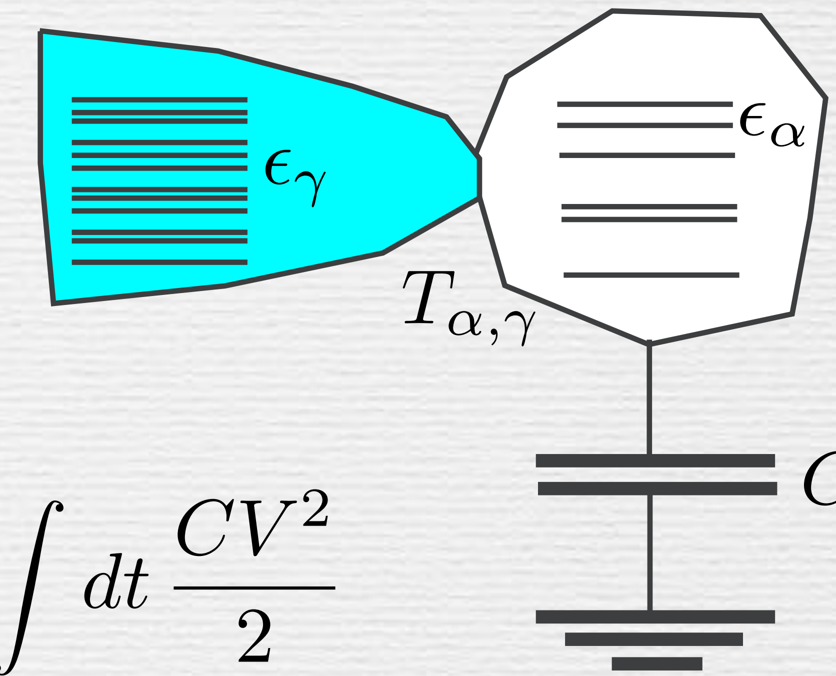
$$i\mathcal{S}_V = \mathrm{tr}\ln\left[i\partial_t - H_{dot}^0 - eV(t) - \Sigma\right] + i\int dt\,\frac{CV^2}{2}$$

$$H_{dot}^0 \equiv \sum_\alpha \epsilon_\alpha\,|\alpha\rangle\langle\alpha|$$

$$\Sigma(t_1,t_2) \equiv T G_{lead}(t_1,t_2) T^\dagger$$

Self-energy due to reservoir

U(1) case



Eliminating V(t)

$$i\mathcal{S}_V = \mathrm{tr}\ln\left[R^{-1}\left\{i\partial_t - H^0_{dot} - eV(t) - \Sigma\right\}R\right] + i\int dt\,\frac{CV^2}{2}$$

$$R(t) = e^{-i\phi(t)} \qquad \dot{\phi}(t) = eV(t)$$

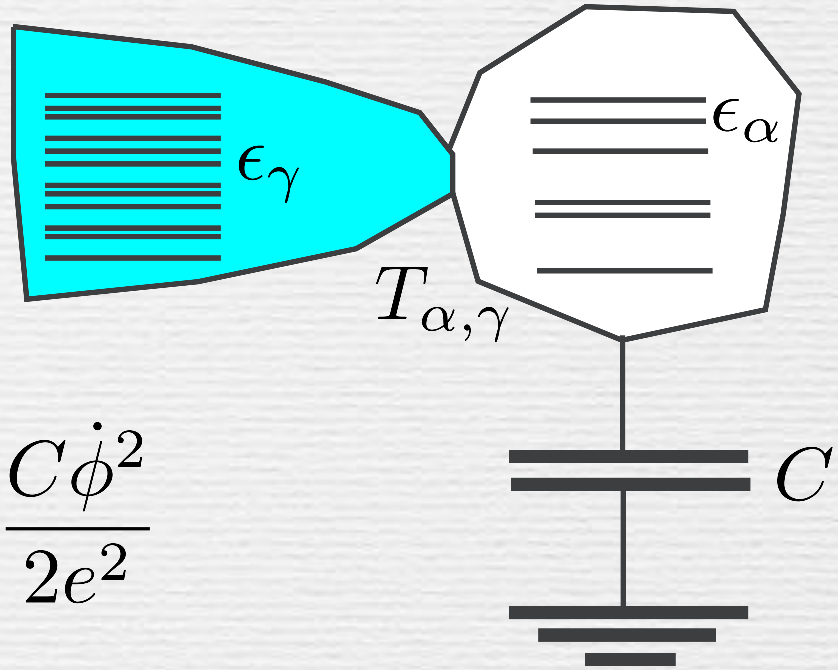
$$i\mathcal{S}_V = \mathrm{tr}\ln\left[i\partial_t - H^0_{dot} - R^{-1}(t_1)\Sigma(t_1,t_2)R(t_2)\right] + i\int dt\,\frac{C\dot{\phi}^2}{2e^2}$$

Expansion in tunneling amplitudes

$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\,\alpha(t_1,t_2)R^{-1}(t_1)R(t_2) + i\int dt\,\frac{C\dot{\phi}^2}{2e^2}$$

$$\alpha(t_1,t_2) \equiv \mathrm{tr}\left[G_{dot}(t_2,t_1)TG_{lead}(t_1,t_2)T^\dagger\right]$$

U(1) case



$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\, \alpha(t_1,t_2) R^{-1}(t_1) R(t_2) + i\int dt\, \frac{C\dot{\phi}^2}{2e^2}$$

$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\, \alpha(t_1,t_2) \cos\left[\phi(t_1) - \phi(t_2)\right] + i\int dt\, \frac{C\dot{\phi}^2}{2e^2}$$

Matsubara

$$\alpha(\tau) = \frac{\pi g}{\sin^2(\pi\tau/\beta)}$$

Tunneling conductance

$$g = \pi \rho_{lead} \rho_{dot} |T|^2$$

AES vs. Caldeira-Leggett (CL) action in mesoscopic physics

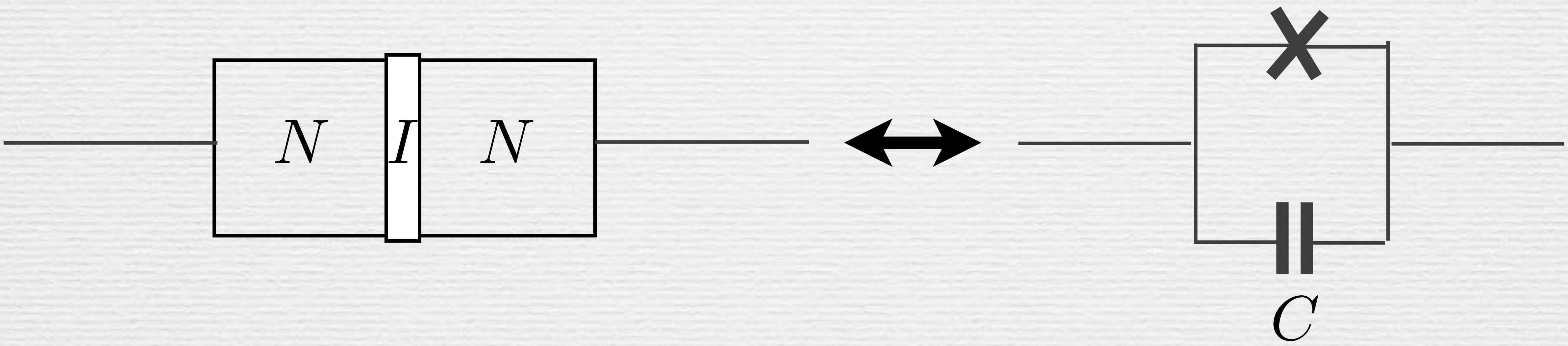
A.O. Caldeira and A.J. Leggett
Phys. Rev. Lett. 46, 211 (1981)

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. 48, 1745 (1982)

AES for tunnel junctions

1) Normal tunnel junction (NIN)

$$R_T = \frac{1}{2g}\frac{2\pi\hbar}{e^2}$$



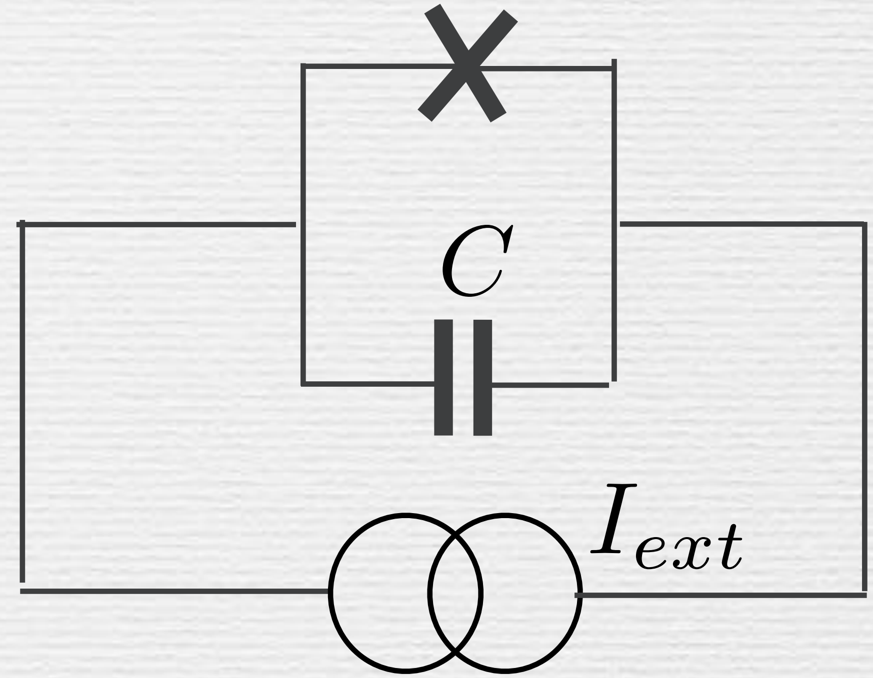
$$i\mathcal{S}_{AES} = i\int dt\,\frac{C\dot{\phi}^2}{2e^2} - \int dt_1 dt_2\,\alpha(t_1,t_2)\,\cos\left[\phi(t_1)-\phi(t_2)\right]$$

$$\dot{\phi}(t) = eV_L(t) - eV_R(t)$$

AES for tunnel junctions

$$R_T = \frac{1}{2g}\frac{2\pi\hbar}{e^2}$$

Normal tunnel junction (NIN)



$$i\mathcal{S}_{AES} = i\int dt\left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e}\right]$$

$$-\int dt_1 dt_2\,\alpha(t_1,t_2)\,\cos\left[\phi(t_1) - \phi(t_2)\right]$$

Langevin eq. of motion

$$C\ddot{\phi} + \frac{\dot{\phi}}{R_T} = I_{ext} + \underbrace{\xi_1\cos\phi + \xi_2\sin\phi}_{\delta I}$$

$$\langle \xi_n \xi_m \rangle = \delta_{n,m}\frac{\hbar\omega}{R_T}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}$$

$$\phi = Vt + \delta\phi$$

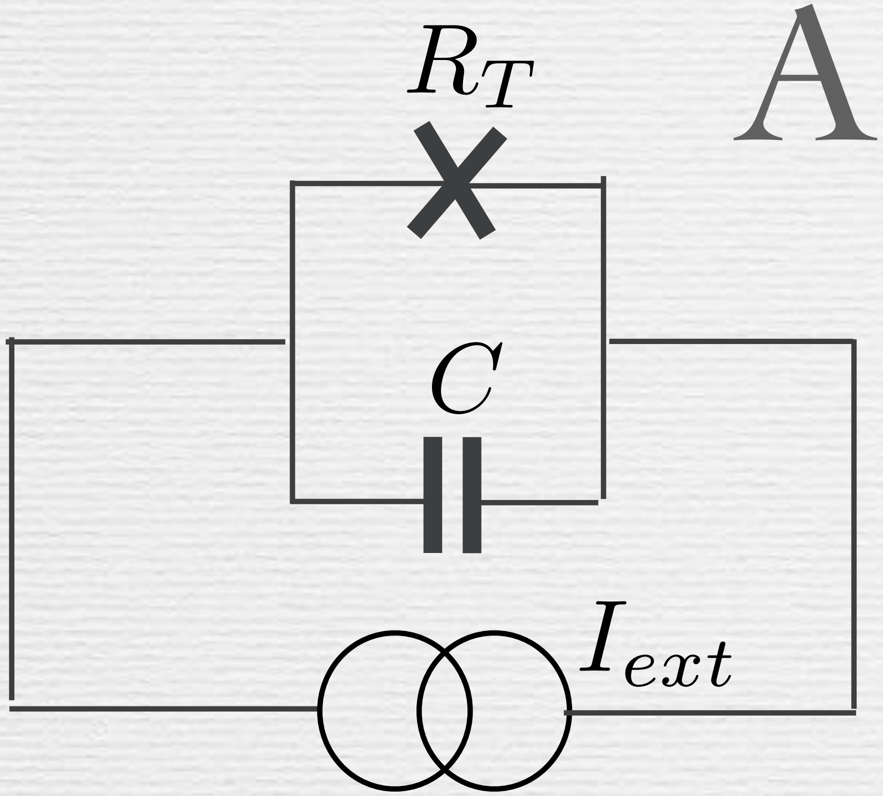
$$V \approx R_T I_{ext}$$

→

shot noise

$$\langle \delta I \delta I \rangle \sim \frac{eV}{R_T} \sim eI$$

AES vs. CL

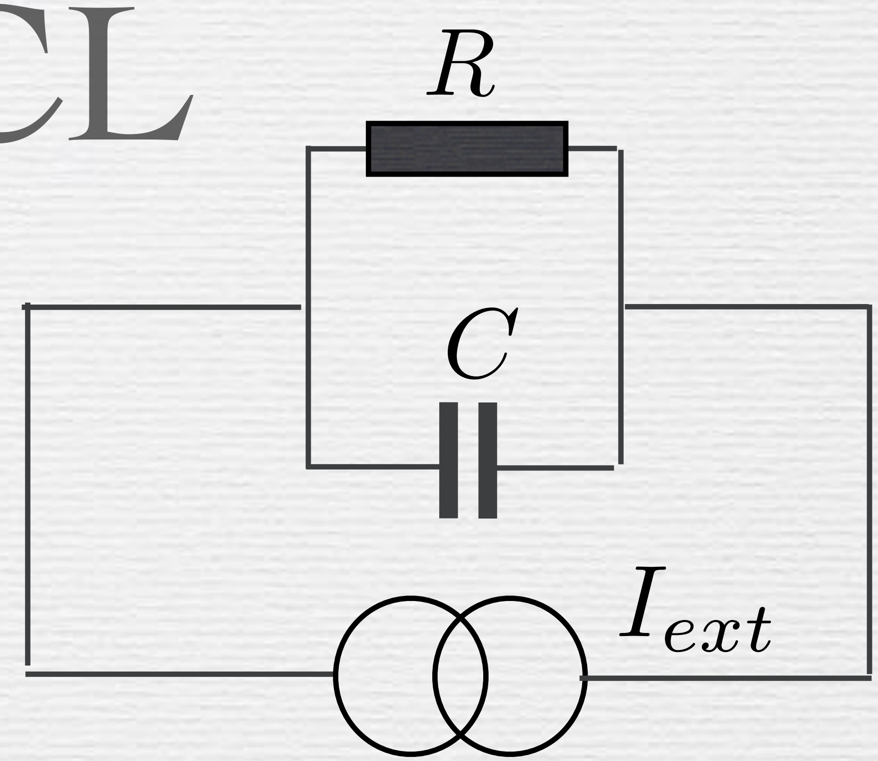


$$i\mathcal{S}_{AES} = i\int dt\left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e}\right]$$

$$-\int dt_1 dt_2\, \alpha(t_1,t_2)\, \cos\left[\phi(t_1) - \phi(t_2)\right]$$

$$C\ddot{\phi} + \frac{\dot{\phi}}{R_T} = I_{ext} + \xi_1 \cos\phi + \xi_2 \sin\phi$$

shot noise



$$i\mathcal{S}_{CL} = i\int dt\left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e}\right]$$

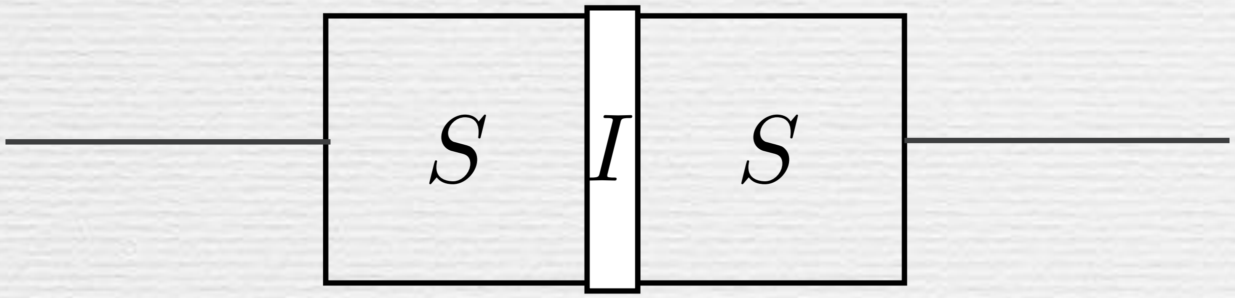
$$+\int dt_1 dt_2\, \alpha(t_1,t_2)\, \frac{\left[\phi(t_1) - \phi(t_2)\right]^2}{2}$$

$$C\ddot{\phi} + \frac{\dot{\phi}}{R} = I_{ext} + \xi$$

no shot noise

AES for tunnel junctions

2) Josephson junction (SIS)



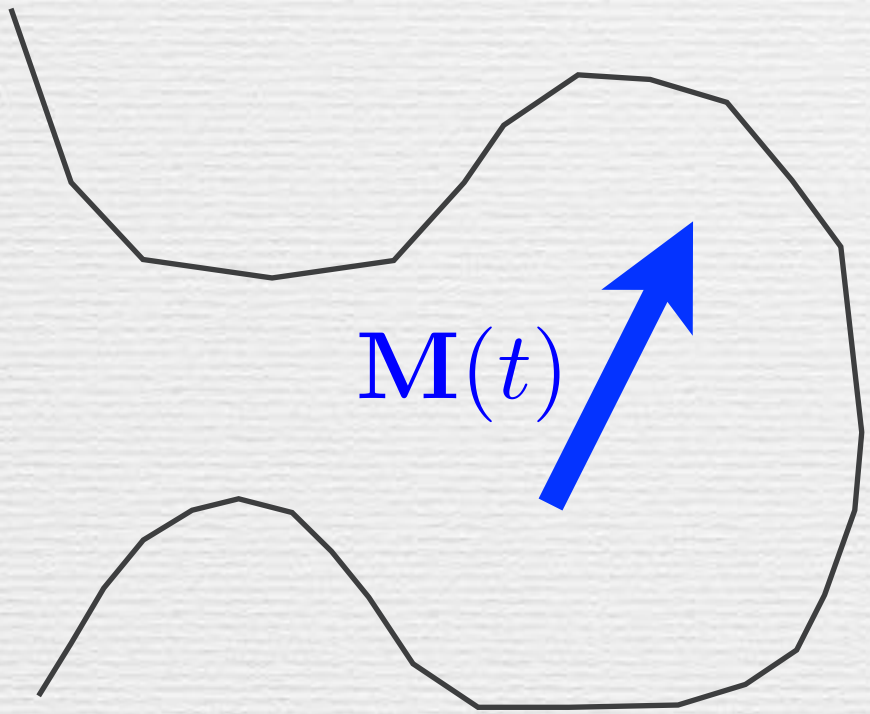
$$i\mathcal{S}_{AES} = i\int dt\,\frac{C\dot{\phi}^2}{2e^2} - \int dt_1 dt_2\,\alpha(t_1,t_2)\,\cos\left[\phi(t_1)-\phi(t_2)\right] - \int dt_1 dt_2\,\beta(t_1,t_2)\,\cos\left[\phi(t_1)+\phi(t_2)\right]$$

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. **48**, 1745-1748 (1982)

Non-Abelian SU(2) case

Open magnetic quantum dot
AES tunnel action

Open dot, "AES" action



$$H = H_{dot} + H_{lead} + H_t$$

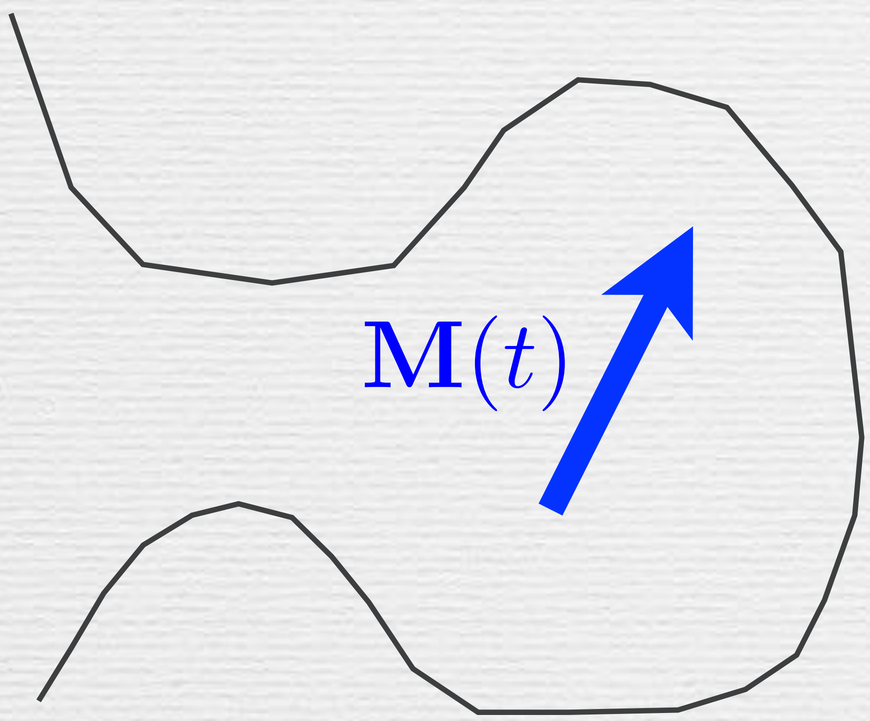
$$H_{dot} = \sum_{\alpha,\sigma} \epsilon_\alpha \psi^\dagger_{\alpha,\sigma} \psi_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

$$H_{lead} = \sum_{\gamma,\sigma} \epsilon_{\gamma,\sigma} c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma}$$

$$H_T = \sum_{\alpha,\gamma,\sigma} T_{\alpha,\gamma} \psi^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + h.c.$$

A. L. Chudnovskiy, J. Swiebodzinski, and A. Kamenev, Phys. Rev. Lett. 101, 066601 (2008)

Open dot, effective action

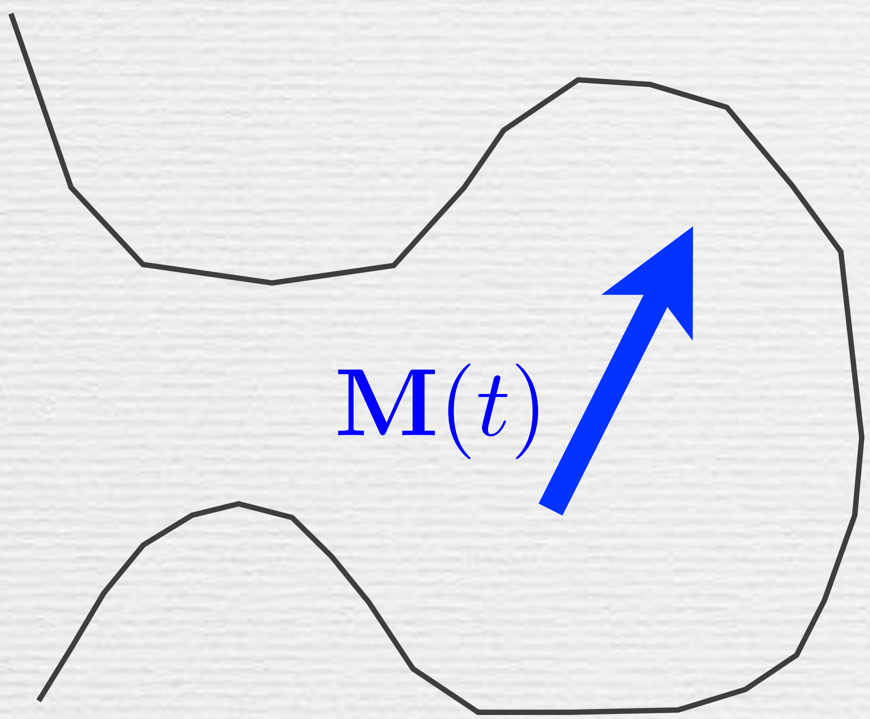


$$\mathcal{S}_M = \mathrm{tr}\,\ln \begin{pmatrix} G_{dot}^{-1} & -T \\ -T^{\dagger} & G_{lead}^{-1} \end{pmatrix} - \oint_K dt\, \frac{M^2}{4J}$$

$$G_{dot}^{-1} = i\partial_t - \epsilon_\alpha - \mathbf{M}(t)\cdot\mathbf{S}$$

$$G_{lead}^{-1} = i\partial_t - \epsilon_\gamma$$

Open dot, effective action



Non-Abelian

$$i\mathcal{S}_M = \mathrm{tr}\ \ln\left[i\partial_t - H^0_{dot} - \mathbf{M}(t)\cdot\mathbf{S} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$H^0_{dot} \equiv \sum_\alpha \epsilon_\alpha\,|\alpha\rangle\langle\alpha|$

$$\Sigma \equiv T G_{lead} T^\dagger$$

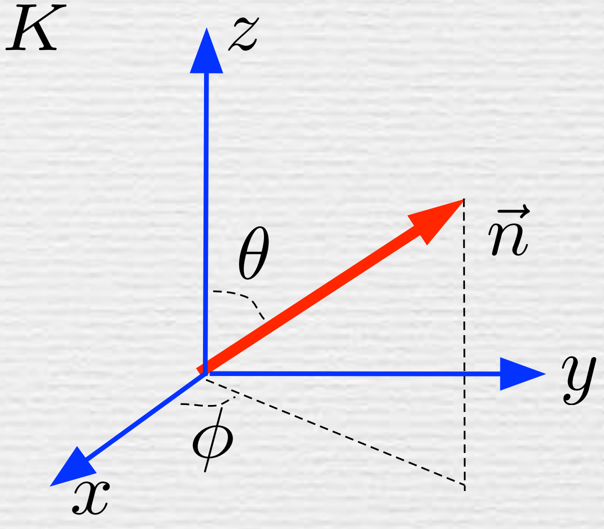
Self-energy due to reservoir

Open dot, rotating frame

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - M(t)\,\vec{n}(t)\cdot\vec{\mathbf{S}} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$$\vec{n}\cdot\vec{\mathbf{S}} = R\,S_z\,R^\dagger \qquad R \in SU(2)/U(1)$$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[\frac{i(\phi-\chi)}{2}\sigma_z\right]$$



$$i\mathcal{S}_\Phi = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger\Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

Geom. vector potential

$$Q \equiv R^\dagger(-i\partial_t)R$$

Rotated tunneling self-energy

Open dot, vector potential

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$$Q \equiv R^\dagger(-i\partial_t)R = Q_\parallel + Q_\perp$$

$$Q_\parallel \equiv \frac{1}{2}\left[\dot\phi(1-\cos\theta) - \dot\chi\right]\sigma_z$$

Berry's phase, gauge dependent

$$Q_\perp \equiv -\frac{1}{2}\left[\dot\theta\,\sigma_y - \dot\phi\sin\theta\,\sigma_x\right]\exp\left[i(\phi-\chi)\,\sigma_z\right]$$

Landau-Zener, neglected

Tunneling expansion, "AES"

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$ Gauge invariant

$$G_0^{-1} = i\partial_t - H^0_{dot} - M\cdot S_z$$

Expansion

$$i\mathcal{S}_M^{Berry} = -\mathrm{tr}\ \left[G_0 Q\right] = iS\oint_K (1-\cos\theta)\dot{\phi}dt$$ Berry phase

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[G_0 R^\dagger \Sigma R\right]$$ Gauge non-invariant

in original U(1) AES

$$R^\dagger(t)R(t') \sim e^{i\left[\varphi(t)-\varphi(t')\right]}$$

V. Ambegaokar, U. Eckern, G. Schön, Phys. Rev. Lett. **48**, 1745-1748 (1982)

Explicit form for non-magnetic lead

$$i\mathcal{S}_M^{AES} = -\int dt_1\, dt_2\, \alpha(t_1 - t_2)\, \mathrm{tr}\left[R(t_1)R^{-1}(t_2)\right]$$

Matsubara

$$\alpha(\tau) = \frac{\pi g}{\sin^2(\pi\tau/\beta)}$$

Tunneling conductance

$$g = \pi\rho_{lead}\rho_{dot}|T|^2$$

$$\mathrm{tr}\left[R(t_1)R^{-1}(t_2)\right] =$$

$$\cos\frac{\theta(t_1)}{2}\cos\frac{\theta(t_2)}{2}\cos\left(\frac{\chi(t_1)-\chi(t_2)}{2}\right)$$

$$+\sin\frac{\theta(t_1)}{2}\sin\frac{\theta(t_2)}{2}\cos\left(\phi(t_1)-\phi(t_2)-\frac{\chi(t_1)-\chi(t_2)}{2}\right)$$

Not gauge invariant

Tunneling expansion, gauge fixing

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right]$$

Gauge invariant expansion

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[(G_0^{-1} - Q)^{-1} R^\dagger \Sigma R\right]$$

Would be nice to choose gauge such that $Q = 0$

$$Q_\parallel \equiv \frac{1}{2}\left[\dot{\phi}(1-\cos\theta) - \dot{\chi}\right]\sigma_z = 0$$

$$\dot{\chi} = \dot{\phi}(1-\cos\theta)$$

Would be nice, but ...

Gauge fixing

$$Q_{\parallel} = 0 \quad \Rightarrow \quad \dot{\chi} = \dot{\phi}(1 - \cos\theta)$$

Would be nice, but impossible

Berry phase different on two contours

$$\dot{\chi}_c(t) = \dot{\phi}_c(t)\,(1 - \cos\theta_c(t)) \quad \Rightarrow \quad Q_{\parallel,c} = 0$$

$$\chi_q(t) = \phi_q(t)\,(1 - \cos\theta_c(t))$$

$$\Rightarrow \quad Q_{\parallel,q} = \frac{1}{2}\sigma_z \sin\theta_c \left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

$$i\mathcal{S}_{WZNW} = iS\int dt\,\sin\theta_c\left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

Keldysh Berry phase action

Semiclassical equations of motion

AES action on Keldysh contour

U. Eckern, G. Schön, V. Ambegaokar, Phys. Rev. B 30, 6419-6431 (1984)

$$i\mathcal{S}_{AES} = -g\int dt_1 dt_2 \,\mathrm{tr}\left[\begin{pmatrix} R_c^\dagger(t_1) & \frac{R_q^\dagger(t_1)}{2}\end{pmatrix}\begin{pmatrix} 0 & \alpha_A \\ \alpha_R & \alpha_K \end{pmatrix}_{(t_1-t_2)}\begin{pmatrix} R_c(t_2) \\ \frac{R_q(t_2)}{2}\end{pmatrix}\right]$$

$g = \pi\rho_{lead}\rho_{dot}|T|^2$ Tunneling conductance

$\alpha_R(\omega) = \omega + symm.part$ $\qquad$ $\alpha_K(\omega) = 2\omega\coth(\omega/2T)$

$$R_c \equiv \frac{R_u + R_d}{2}$$

$$R_q \equiv R_u - R_d$$