Dissipative magnetic dynamics

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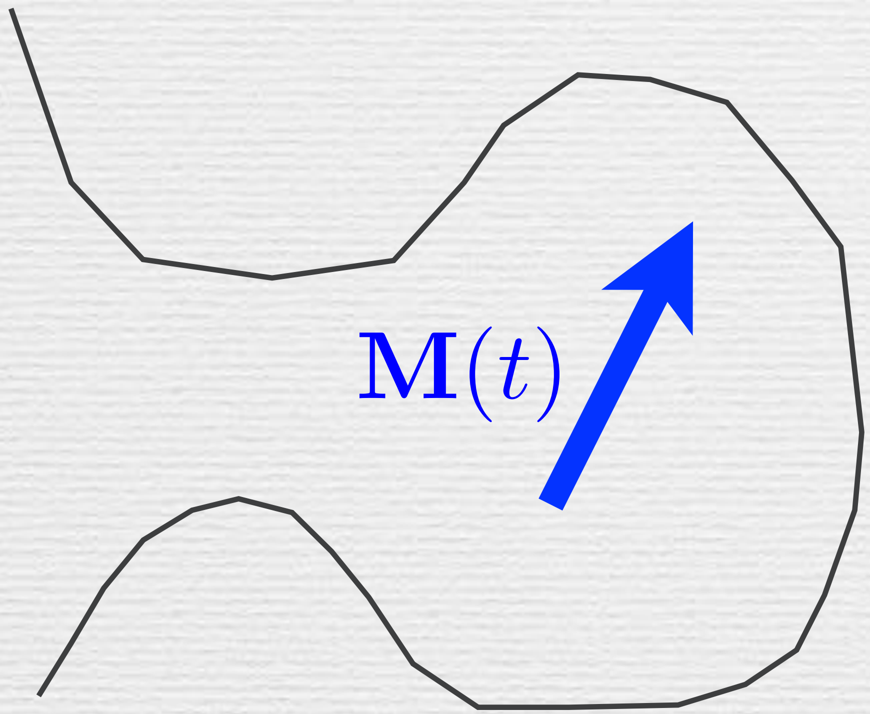
Lecture 2

Plan

1. Ambegaokar-Eckern-Schön (AES) effective action for SU(2) - LLG-Langevin equations
2. Spin transfer torque
3. Strong non-equilibrium effects

Reminder about Lecture 1

Open magnetic dot “AES” action



$$H = H_{dot} + H_{lead} + H_t$$

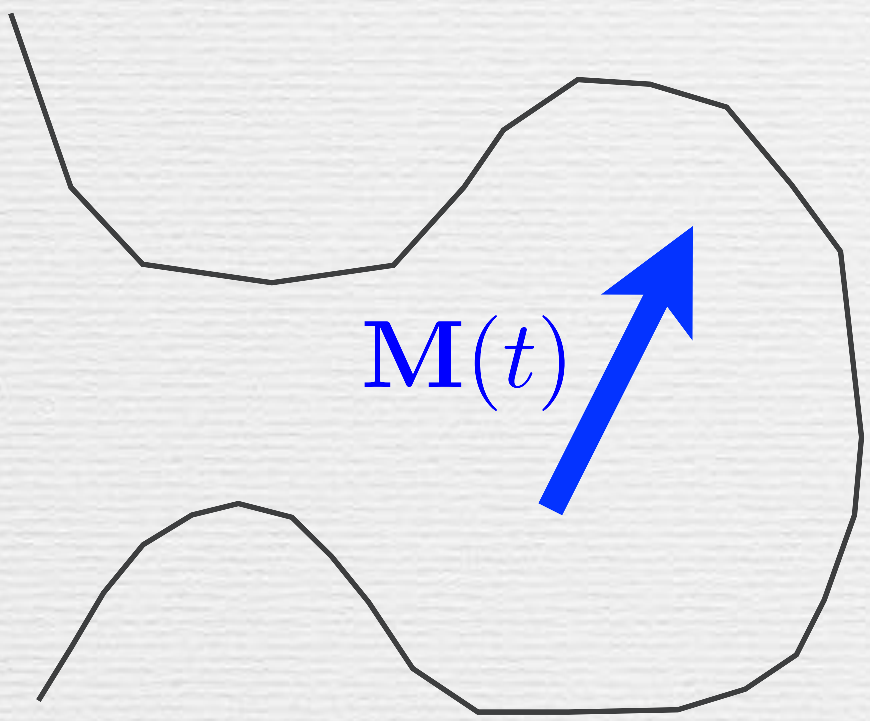
$$H_{dot} = \sum_{\alpha,\sigma} \epsilon_\alpha \psi^\dagger_{\alpha,\sigma} \psi_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

$$H_{lead} = \sum_{\gamma,\sigma} \epsilon_{\gamma,\sigma} c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma}$$

$$H_T = \sum_{\alpha,\gamma,\sigma} T_{\alpha,\gamma} \psi^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + h.c.$$

A. L. Chudnovskiy, J. Swiebodzinski, and A. Kamenev, Phys. Rev. Lett. 101, 066601 (2008)

Open dot, effective action



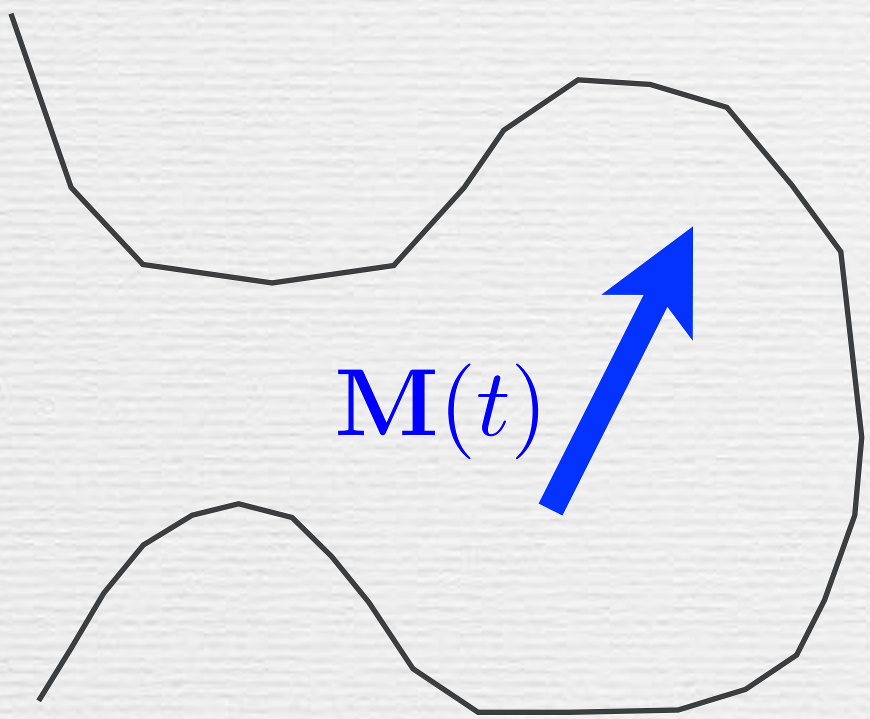
$$\mathcal{S}_M = \mathrm{tr}\,\ln\begin{pmatrix} G_{dot}^{-1} & -T \\ -T^\dagger & G_{lead}^{-1} \end{pmatrix} - \oint_K dt\,\frac{M^2}{4J}$$

$$G_{dot}^{-1} = i\partial_t - \epsilon_\alpha - \mathbf{M}(t)\cdot\mathbf{S}$$

$$G_{lead}^{-1} = i\partial_t - \epsilon_\gamma$$

Assume $|\vec{M}| = const. > 0$
mesoscopic Stoner
or ferromagnet

Open dot, effective action



Non-Abelian

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - \mathbf{M}(t)\cdot\mathbf{S} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$

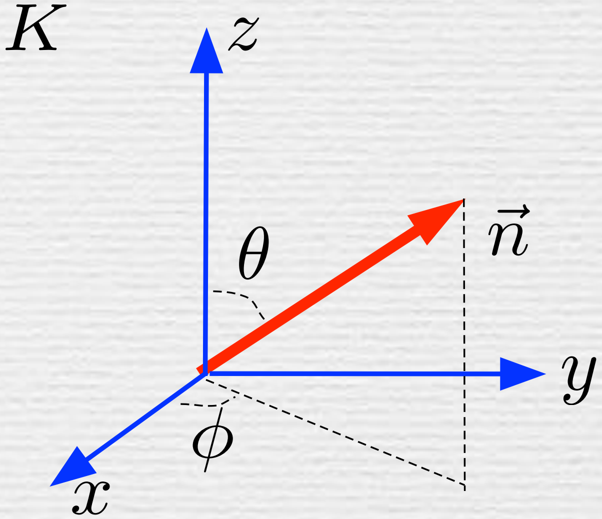
$H^0_{dot} \equiv \sum_\alpha \epsilon_\alpha\,|\alpha\rangle\langle\alpha|$

$$\Sigma \equiv T G_{lead} T^\dagger$$

Self-energy due to reservoir

Open dot, rotating frame

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - M(t)\,\vec{n}(t)\cdot\vec{\mathbf{S}} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$



$$\vec{n}\cdot\vec{\mathbf{S}} = R\,S_z\,R^\dagger \qquad R \in SU(2)/U(1)$$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[\frac{i(\phi-\chi)}{2}\sigma_z\right]$$

$$i\mathcal{S}_\Phi = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger\Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

Geom. vector potential

$$Q \equiv R^\dagger(-i\partial_t)R$$

Rotated tunneling self-energy

Open dot, vector potential

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$$Q \equiv R^\dagger(-i\partial_t)R = Q_\parallel + Q_\perp \qquad Q_\parallel \equiv \frac{1}{2}\left[\dot\phi(1-\cos\theta) - \dot\chi\right]\sigma_z$$

Berry's phase, gauge dependent

Tunneling expansion, “AES”

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$ Gauge invariant

$$G_0^{-1} = i\partial_t - H^0_{dot} - M\cdot S_z$$

Expansion

$$i\mathcal{S}_M^{Berry} = -\mathrm{tr}\;[G_0 Q] = iS\oint_K (1-\cos\theta)\dot{\phi}dt$$ Berry phase

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[G_0 R^\dagger \Sigma R\right]$$ Gauge non-invariant

Tunneling expansion, gauge fixing

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right]$$

Gauge invariant expansion

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[(G_0^{-1} - Q)^{-1} R^\dagger \Sigma R\right]$$

Would be nice to choose gauge such that $Q = 0$

$$Q_\parallel \equiv \frac{1}{2}\left[\dot{\phi}(1-\cos\theta) - \dot{\chi}\right]\sigma_z = 0$$

$$\dot{\chi} = \dot{\phi}(1-\cos\theta)$$

Would be nice, but ...

Gauge fixing

$$Q_{\parallel} = 0 \quad \Rightarrow \quad \dot{\chi} = \dot{\phi}(1 - \cos\theta)$$

Would be nice, but impossible

Berry phase different on two contours

$$\dot{\chi}_c(t) = \dot{\phi}_c(t)\,(1 - \cos\theta_c(t)) \quad \Rightarrow \quad Q_{\parallel,c} = 0$$

$$\chi_q(t) = \phi_q(t)\,(1 - \cos\theta_c(t))$$

$$\searrow \quad Q_{\parallel,q} = \frac{1}{2}\sigma_z \sin\theta_c \left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

$$i\mathcal{S}_{WZNW} = iS\int dt\,\sin\theta_c\left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

Keldysh Berry phase action

SU(2)
Semiclassical equations of motion-Landau-Lifshitz-Gilbert-Langevin

AES action on Keldysh contour

$$i\mathcal{S}_{AES} = -g \int dt_1 dt_2 \,\mathrm{tr}\left[\begin{pmatrix} R_c^\dagger(t_1) & \frac{R_q^\dagger(t_1)}{2} \end{pmatrix} \begin{pmatrix} 0 & \alpha_A \\ \alpha_R & \alpha_K \end{pmatrix}_{(t_1 - t_2)} \begin{pmatrix} R_c(t_2) \\ \frac{R_q(t_2)}{2} \end{pmatrix} \right]$$

$g = \pi \rho_{lead} \rho_{dot} |T|^2$ Tunneling conductance

$\alpha_R(\omega) = \omega + symm.part$ $\alpha_K(\omega) = 2\omega \coth(\omega/2T)$

$$R_c \equiv \frac{R_u + R_d}{2}$$

$$R_q \equiv R_u - R_d$$

Equations of motion

$$i\mathcal{S}_{total} \equiv i\mathcal{S}_{WZNW} + i\mathcal{S}_B + i\mathcal{S}^R_{AES} + i\mathcal{S}^K_{AES}$$

$$i\mathcal{S}_{WZNW} = iS \int dt\, \sin\theta_c \left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right] \qquad i\mathcal{S}_B = -iS\gamma\, B \int dt\, \theta_q\, \sin\theta_c$$

$$i\mathcal{S}^R_{AES} = -2ig \int dt_1\, dt_2\, \mathrm{Im}\,\alpha_R(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^c_n(t_2)$$

$$i\mathcal{S}^K_{AES} = -\frac{g}{2} \int dt_1\, dt_2\; \alpha_K(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^q_n(t_2)$$

$$A_0 \equiv \cos\left[\tfrac{\theta}{2}\right]\, \cos\left[\tfrac{\chi}{2}\right],\; A_x \equiv \sin\left[\tfrac{\theta}{2}\right] \sin\left[\phi - \tfrac{\chi}{2}\right],$$
$$A_y \equiv -\sin\left[\tfrac{\theta}{2}\right] \cos\left[\phi - \tfrac{\chi}{2}\right],\; A_z \equiv -\cos\left[\tfrac{\theta}{2}\right]\, \sin\left[\tfrac{\chi}{2}\right]$$

Landau-Lifshitz-Gilbert equation

$$i\mathcal{S}_{total} \equiv i\mathcal{S}_{WZNW} + i\mathcal{S}_B + i\mathcal{S}^R_{AES} + i\mathcal{S}^K_{AES}$$

$$\sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\,\dot{\theta} = 0$$

$$\vec{n} \equiv \frac{\vec{S}}{S} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\dot{\theta} + \frac{g}{S}\,\sin\theta\dot{\phi} = 0$$

$$\frac{d\vec{n}}{dt} = \vec{B}\times\vec{n} + \frac{g}{S}\,\vec{n}\times\frac{d\vec{n}}{dt}$$

Keldysh part - Langevin terms

$$i\mathcal{S}^K_{AES} = -\frac{g}{2}\int dt_1\, dt_2\ \alpha^K_{AES}(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^q_n(t_2)$$

$$e^{i\mathcal{S}^K_{AES}} = \int \left(\prod_{n=0,x,y,z} D\xi_n\right) \exp\left[\int dt \left\{ i \sum_{n=0,x,y,z} \xi_n A^q_n \right\} + i\mathcal{S}_\xi \right]$$

$$i\mathcal{S}_\xi = -\frac{1}{2}\sum_n \int dt_1 dt_2 \left[\alpha^K_{AES}\right]^{-1}_{(t_1-t_2)} \xi_n(t_1)\xi_n(t_2)$$

$$\Downarrow$$

$$i\mathcal{S}_{total} \equiv i\mathcal{S}_B + i\mathcal{S}_{WZNW} + i\mathcal{S}^R_{AES} + \int dt \sum_n i\xi_n A^q_n$$

$$\langle \xi_n(t_1)\xi_m(t_2)\rangle = \delta_{nm}\, \alpha^K_{AES}(t_1 - t_2)$$

Landau-Lifshitz-Gilbert-Langevin equation

$$\dot{\theta} + \frac{g}{S}\sin\theta\dot{\phi} = \frac{1}{S}\eta_\theta \qquad \sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\dot{\theta} = \frac{1}{S}\eta_\phi$$

$$\begin{aligned}
\eta_\theta = & \quad \frac{1}{2}\cos\frac{\theta}{2}\left[\xi_x\cos\left(\phi - \frac{\chi}{2}\right) + \xi_y\sin\left(\phi - \frac{\chi}{2}\right)\right] \\
& - \frac{1}{2}\sin\frac{\theta}{2}\left[\xi_z\cos\frac{\chi}{2} + \xi_0\sin\frac{\chi}{2}\right] , \\
\eta_\phi = & - \frac{1}{2}\cos\frac{\theta}{2}\left[\xi_x\sin\left(\phi - \frac{\chi}{2}\right) - \xi_y\cos\left(\phi - \frac{\chi}{2}\right)\right] \\
& - \frac{1}{2}\sin\frac{\theta}{2}\left[\xi_z\sin\frac{\chi}{2} - \xi_0\cos\frac{\chi}{2}\right]
\end{aligned}$$

$$\langle\xi_n\xi_m\rangle_\omega = 2g\,\omega\coth\frac{\omega}{2T}\,\delta_{n,m} \qquad n, m = 0, x, y, z$$

Gauge fixing $\dot{\chi} = \dot{\phi}(1 - \cos\theta)$

AES vs. Caldeira-Leggett

Usual LLG-Langevin equation

W. F. Brown, Phys. Rev. 130, 1677 (1963).

$$\frac{d\vec{n}}{dt} = \left(\vec{B} + \delta\vec{B}\right) \times \vec{n} + \alpha\,\vec{n} \times \frac{d\vec{n}}{dt}$$

$$\langle \delta B_n \delta B_m \rangle_\omega \propto \alpha T\, \delta_{n,m}$$

Three independent Langevin variables
not four !!!

What is the microscopic theory?

Caldeira - Leggett situation

$$\delta H = \vec{S}\vec{X} \equiv -J\vec{S} \cdot \frac{1}{2}\sum_{\alpha,\beta} c^\dagger_{\alpha,\sigma_1}\vec{\sigma}_{\sigma_1\sigma_2}c_{\beta,\sigma_2}$$

Large spin $\vec{S}$ interacting with a bath $\vec{S} \neq \frac{1}{2}\sum_{\alpha,\beta} c^\dagger_{\alpha,\sigma_1}\vec{\sigma}_{\sigma_1\sigma_2}c_{\beta,\sigma_2}$

Caldeira-Leggett action

$$\mathcal{S}_{diss} = \int_0^\beta d\tau_1 \int_0^\beta d\tau_2\, \alpha(\tau_1 - \tau_2)\, \vec{S}(\tau_1) \cdot \vec{S}(\tau_2)$$

$$\alpha(\tau_1 - \tau_2) \sim \langle X(\tau_1) X(\tau_2) \rangle$$

↓

$$\frac{d\vec{n}}{dt} = \left(\vec{B} + \delta\vec{B}\right) \times \vec{n} + S\,\alpha\,\vec{n} \times \frac{d\vec{n}}{dt} \qquad \langle \delta B_n \delta B_m \rangle_\omega \propto \alpha T\, \delta_{n,m}$$

AES vs. Caldeira - Leggett

AES

$$\dot{\theta} + \frac{g}{S} \sin\theta \dot{\phi} = \frac{1}{S} \eta_\theta \qquad \sin\theta \left(\dot{\phi} - B \right) - \frac{g}{S} \dot{\theta} = \frac{1}{S} \eta_\phi$$

$$\eta_\theta = \frac{1}{2} \cos\frac{\theta}{2} \left[\xi_x \cos\left(\phi - \frac{\chi}{2}\right) + \xi_y \sin\left(\phi - \frac{\chi}{2}\right) \right] - \frac{1}{2} \sin\frac{\theta}{2} \left[\xi_z \cos\frac{\chi}{2} + \xi_0 \sin\frac{\chi}{2} \right]$$

$$\eta_\phi = -\frac{1}{2} \cos\frac{\theta}{2} \left[\xi_x \sin\left(\phi - \frac{\chi}{2}\right) - \xi_y \cos\left(\phi - \frac{\chi}{2}\right) \right] - \frac{1}{2} \sin\frac{\theta}{2} \left[\xi_z \sin\frac{\chi}{2} - \xi_0 \cos\frac{\chi}{2} \right]$$

$$\langle \eta_n \eta_m \rangle_\omega \propto gT\, \delta_{n,m} \text{ for } T \gg \omega$$

Caldeira-Leggett

$$\dot{\theta} + S\alpha \sin\theta \dot{\phi} = \eta_\theta \qquad \sin\theta \left(\dot{\phi} - B \right) - S\alpha\, \dot{\theta} = \eta_\phi$$

$$\eta_\theta = \frac{1}{2} \left(-\xi_x \sin\phi + \xi_y \cos\phi \right)$$

$$\eta_\phi = \frac{\sin\theta}{2} \xi_z - \frac{\cos\theta}{2} \left(\xi_x \cos\phi + \xi_y \sin\phi \right)$$

$$\langle \eta_n \eta_m \rangle_\omega \propto \alpha T\, \delta_{n,m} \text{ for } T \gg \omega$$

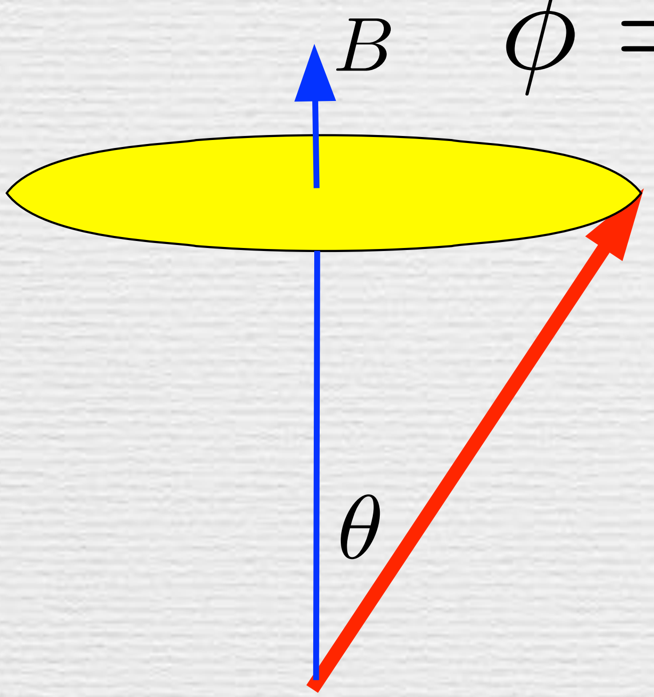
Geometric Langevin terms

Geometric Langevin terms

For example

$$\xi_x \cos\left(\phi - \frac{\chi}{2}\right)$$

$$\langle \xi_n \xi_m \rangle_\omega = 2g\,\omega \coth \frac{\omega}{2T}\,\delta_{n,m}$$



$$\phi = Bt$$

gauge fixing

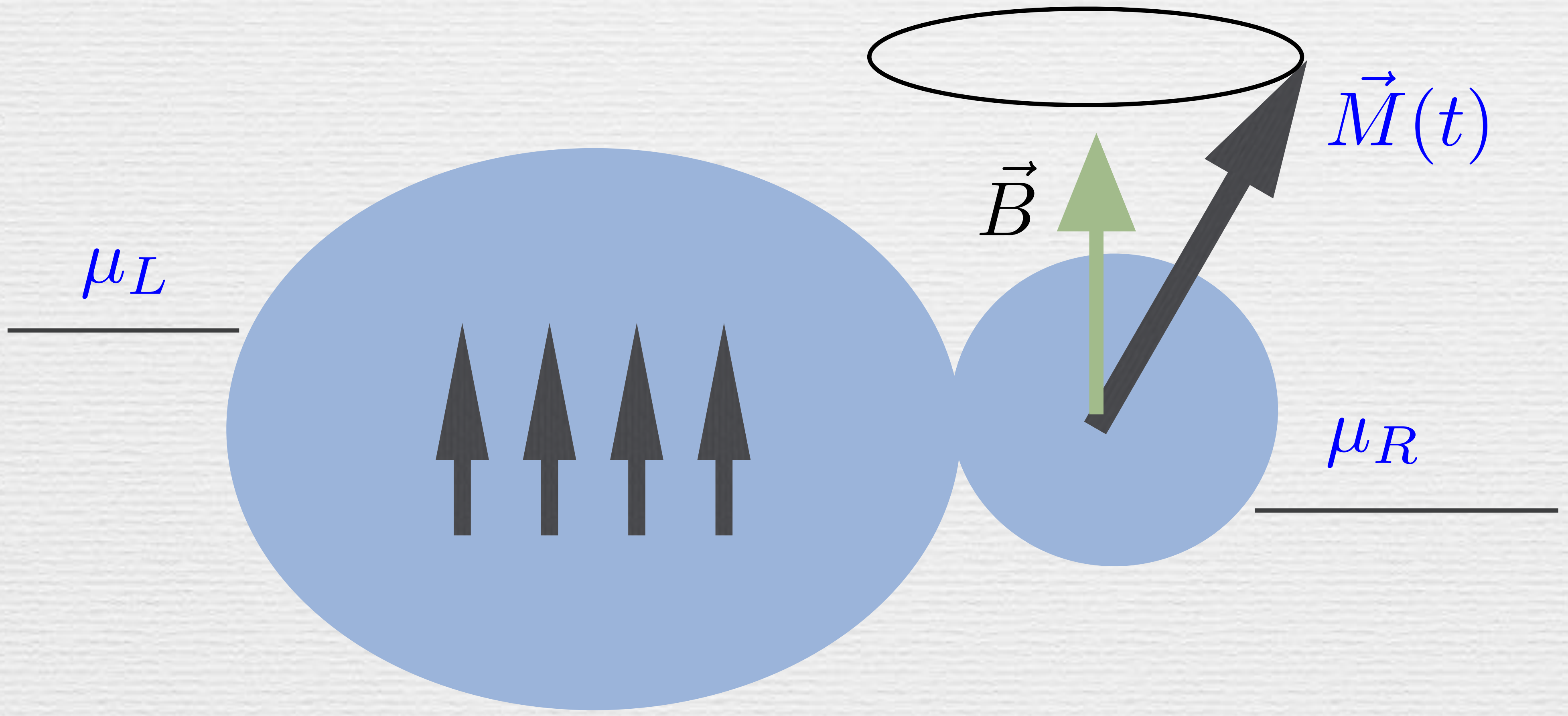
$$\dot\chi = \dot\phi(1-\cos\theta) \rightarrow \chi = Bt(1-\cos\theta)$$

$$\xi_x \cos\left(B\,t\,\cos^2\frac{\theta}{2}\right)$$

noise at $\omega = B\cos^2(\theta/2)$ picked up

Spin transfer torque

Spin transfer torque



AES action

$U = e^{-i\psi} R$

U(1) X SU(2)

$$i\mathcal{S}_{AES} = -\oint dt \oint dt' \; \mathrm{tr}\left[G_{d,z}(t-t') U^{\dagger}(t') \, \Sigma \, (t'-t) U(t) \right]$$

$$U = A_{\uparrow\uparrow}\sigma_{\uparrow} + A_{\downarrow\downarrow}\sigma_{\downarrow} + A_{\downarrow\uparrow}\sigma_{+} + A_{\uparrow\downarrow}\sigma_{-}$$

$$A_{\uparrow,\uparrow} = \cos\frac{\theta}{2}\, e^{i(-\frac{\chi}{2}-\psi)}$$

$$A_{\downarrow,\downarrow} = \cos\frac{\theta}{2}\, e^{i(\frac{\chi}{2}-\psi)}$$

$$A_{\downarrow,\uparrow} = -\sin\frac{\theta}{2}\, e^{i(-\phi+\frac{\chi}{2}-\psi)}$$

$$A_{\uparrow,\downarrow} = \sin\frac{\theta}{2}\, e^{i(\phi-\frac{\chi}{2}-\psi)}$$

U(1) phase -
voltage + counting field

AES action

$$i\mathcal{S}^{R}_{AES} = -i\sum_{\sigma\sigma'} g_{\sigma\sigma'} \int dt\ dt'\ \mathrm{Im}\left[A^{c}_{\sigma\sigma'}(t')\ \alpha^{R}(t-t')\ A^{q*}_{\sigma\sigma'}(t)\right]$$

$$i\mathcal{S}^{K}_{AES} = -\frac{1}{4}\sum_{\sigma\sigma'} g_{\sigma\sigma'} \int dt\ dt'\ A^{q*}_{\sigma\sigma'}(t')\ \alpha^{K}(t-t')\ A^{q}_{\sigma\sigma'}(t)$$

$$\alpha^{R}(\omega) - \alpha^{A}(\omega) = 2\omega$$

$$\alpha^{K}(\omega) = 2\omega \coth\frac{\omega}{2T}$$

$$g_{\sigma\sigma'} = 2\pi |t_l|^2 \rho^{\sigma}_{dot}\rho^{\sigma'}_{lead}$$

Spin dependent tunneling conductances

Details

$$i\mathcal{S} = \mathrm{tr}\ln\Big[-i\Big(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2}\sigma_z}_{G_{d,z}^{-1}} - Q - R^\dagger \Sigma R\Big)\Big] + i\mathcal{S}_B$$

$$G_{d,z}(\epsilon) = \begin{pmatrix} -2\pi i\,\delta(\epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z)\,F_d(\epsilon) & \frac{1}{\epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z + i0} \\ \frac{1}{\epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z - i0} & 0 \end{pmatrix}$$

$$\Sigma_\sigma(\epsilon) \approx \begin{pmatrix} 0 & i\Gamma_l^\sigma \\ -i\Gamma_l^\sigma & -2i\Gamma_l^\sigma\,F_l(\epsilon) \end{pmatrix}$$

$F(\epsilon) \equiv 1 - 2n(\epsilon)$
distribution func.

Γ_l^σ spin-resolved level width

Variation of the action → EOM

$$i\mathcal{S} = i\mathcal{S}_{\mathrm{WZNW}} + i\mathcal{S}_{B} + i\mathcal{S}_{\mathrm{AES}}$$

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = -\mathbf{B}\times\mathbf{M} - \alpha(\theta)\frac{\mathbf{M}}{M_0}\times\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} + \mathbf{M}\times\left(\frac{\mathbf{I}_s}{S}\times\frac{\mathbf{M}}{M_0}\right)$$

Gilbert-damping [1]: $\alpha(\theta) = \dfrac{\tilde{g}(\theta)}{S}$

Spin-torque current [2]: $I_s = g_s V$

Charge current [3]: $I = 4g(\theta)V - g_s \sin^2\theta\dot{\phi}$

[1] A. L. Chudnovskiy, et al. PRL 101 066601 (2008).

[2] J. C. Slonczewski, JMMM 159, L1 (1996). & L. Berger, Phys. Rev. B 54, 9353 (1996)

[3] L. Berger, Phys. Rev. B 59, 11465 (1998), Y. Tserkovnyak et al., Phys. Rev. B 78, 020401(R) (2008)

Equation of Motion - LLG+Slonczewski

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = -\mathbf{B}\times\mathbf{M} - \alpha(\theta)\frac{\mathbf{M}}{M_0}\times\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} + \mathbf{M}\times\left(\frac{\mathbf{I}_s}{S}\times\frac{\mathbf{M}}{M_0}\right)$$

Gilbert-damping $\alpha(\theta) = \frac{\tilde{g}(\theta)}{S}$

Spin-torque current $I_s = g_s V$

Charge current $I = 4g(\theta)V - g_s \sin^2\theta\dot{\phi}$

Conductances

$$\tilde{g}(\theta) = \frac{\sin^2(\frac{\theta}{2})}{4}(g_{\uparrow\uparrow} + g_{\downarrow\downarrow}) + \frac{\cos^2(\frac{\theta}{2})}{4}(g_{\uparrow\downarrow} + g_{\downarrow\uparrow})$$

$$g_s = \frac{1}{4}(g_{\uparrow\uparrow} - g_{\downarrow\downarrow} - g_{\uparrow\downarrow} + g_{\downarrow\uparrow})$$

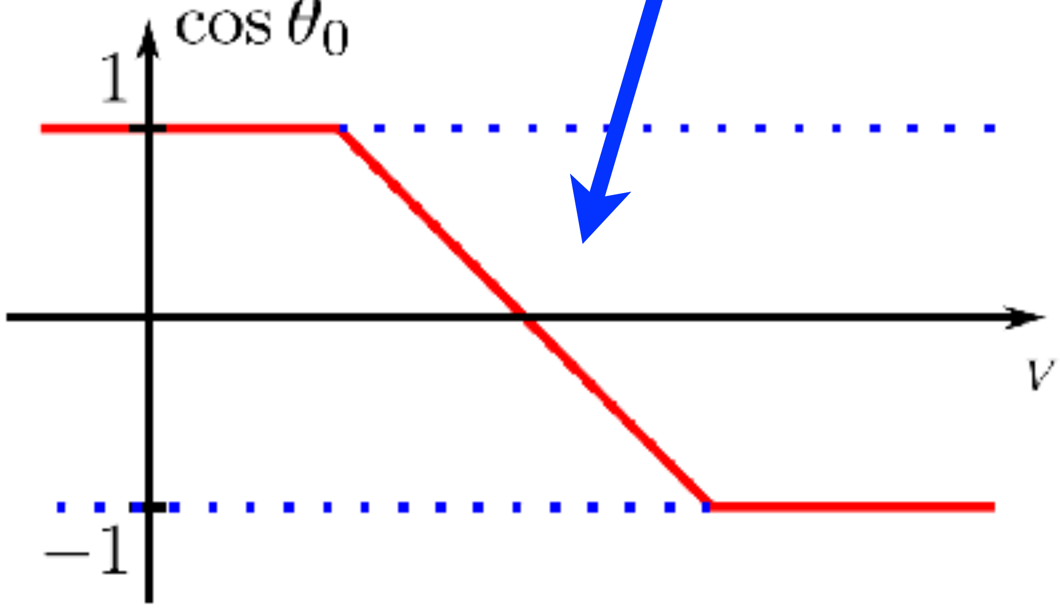
$$g_{\sigma\sigma'} = 2\pi|t_l|^2\rho^{\sigma}_{dot}\rho^{\sigma'}_{lead}$$

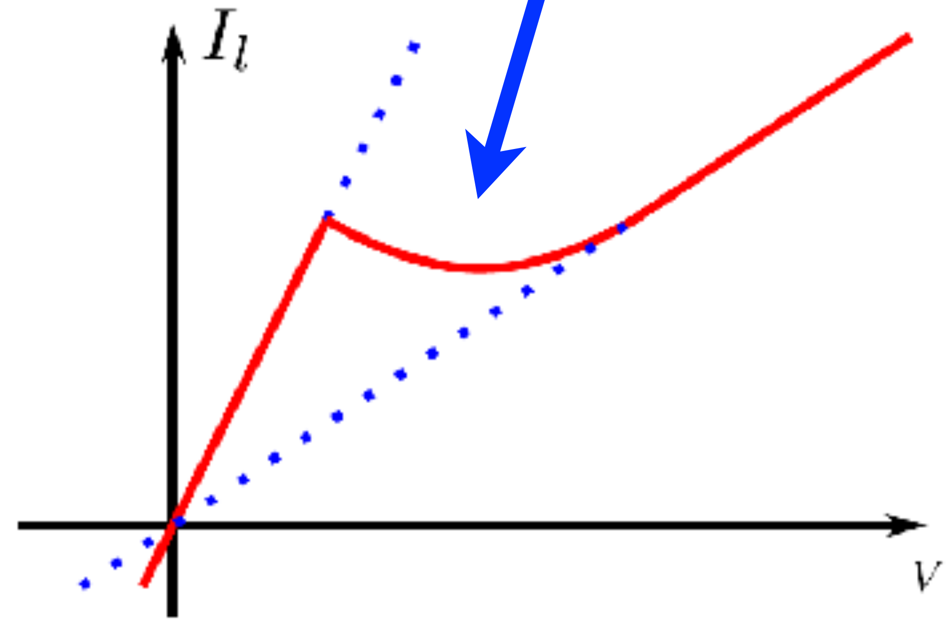
$$g(\theta) = \frac{\cos^2(\frac{\theta}{2})}{4}(g_{\uparrow\uparrow} + g_{\downarrow\downarrow}) + \frac{\sin^2(\frac{\theta}{2})}{4}(g_{\uparrow\downarrow} + g_{\downarrow\uparrow})$$

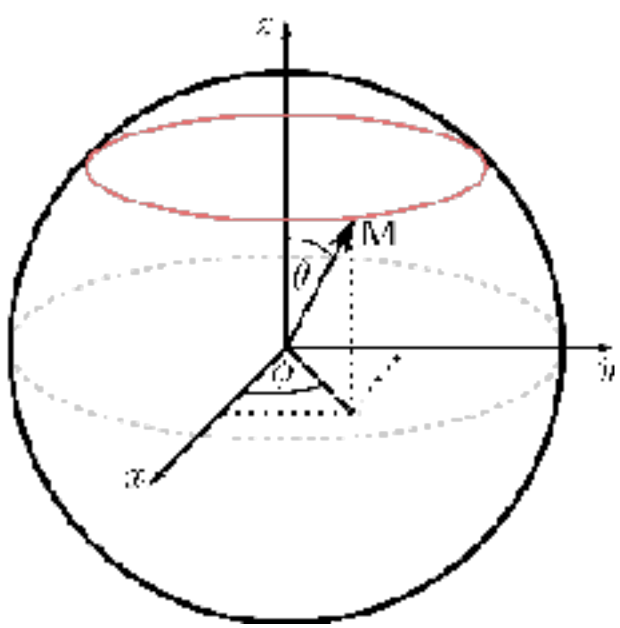
A. L. Chudnovskiy, J. Swiebodzinski, and A. Kamenev, Phys. Rev. Lett. 101, 066601 (2008)

$$\dot{\theta} = -\frac{\sin\theta}{1+\alpha^2}\underbrace{[\alpha(\theta)B + g_s V/S]}$$

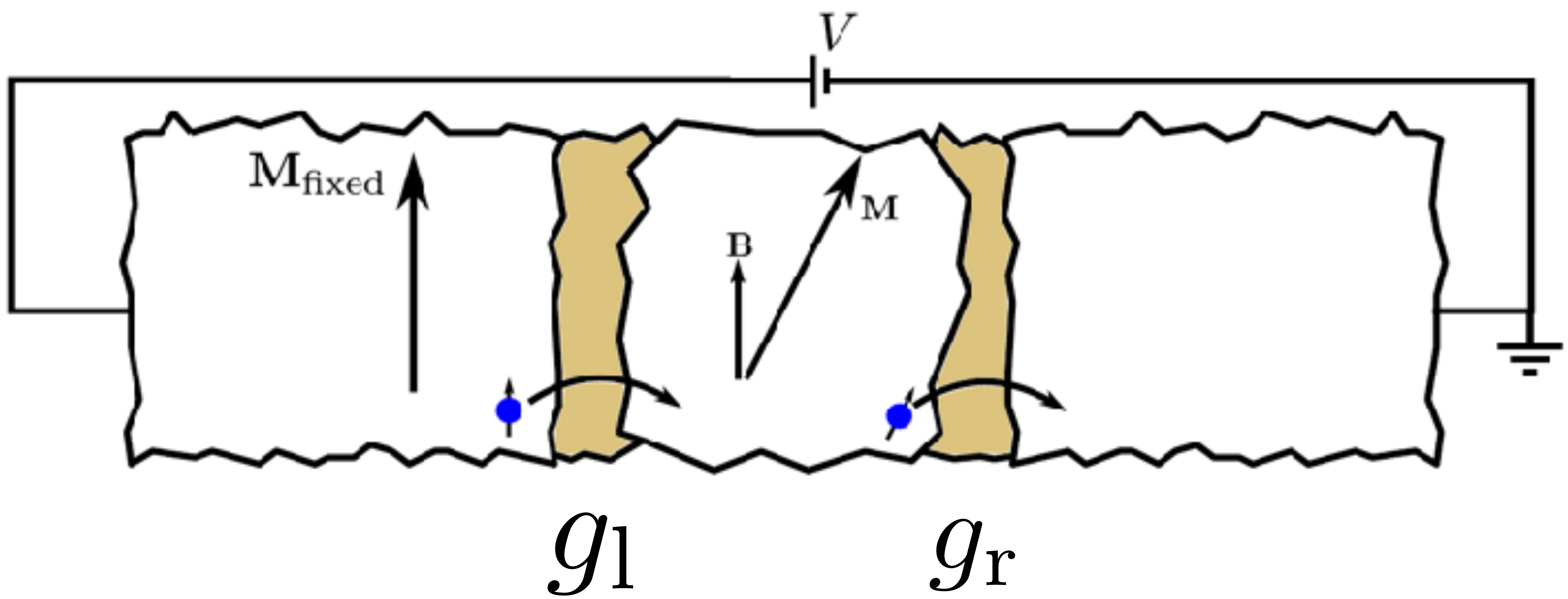
$$I_l = 4g(\theta)V - g_s \sin^2\theta\dot{\phi}$$





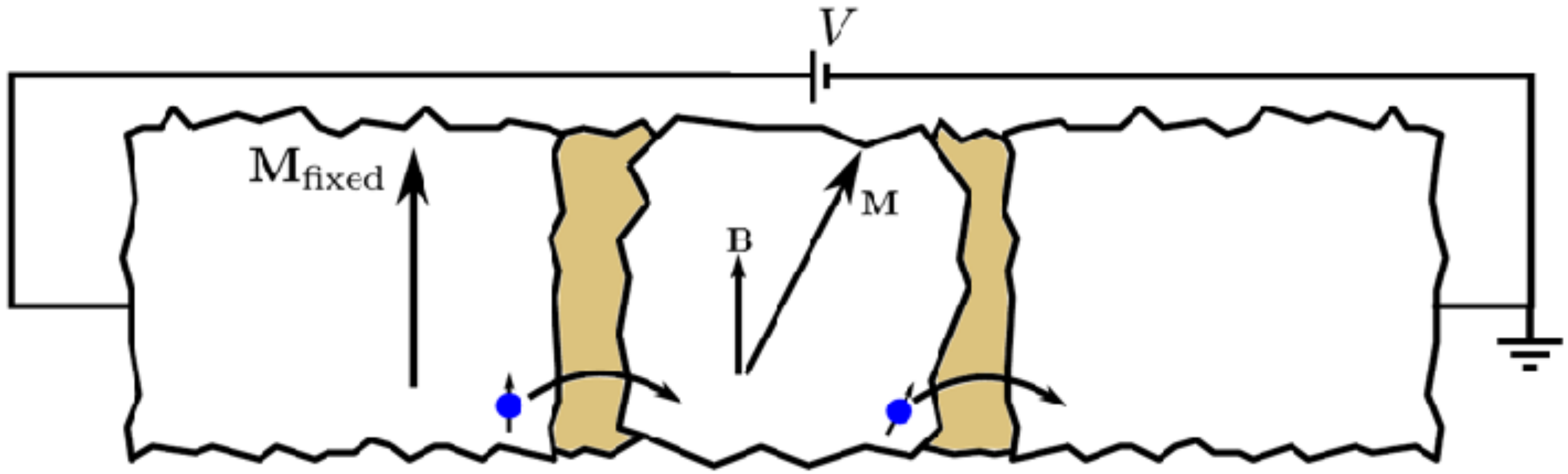


Strong non-equilibrium



The electron distribution on the dot strongly affected by driving!

This makes a drastic change even in the limit of $g_r \gg g_l$



$$i\mathcal{S}[\mathbf{M}, V_d] = \mathrm{tr}\ln\left[-i\left(G_{d,0}^{-1} + \mathbf{M}\frac{\boldsymbol{\sigma}}{2} - V_d - \Sigma\right)\right] - i\overbrace{\oint_K dt\,\frac{(\mathbf{M}-\mathbf{B})^2}{4J}}^{-\mathcal{S}_B} + i\overbrace{\oint_K dt\left(\frac{CV_d^2}{2} + V_d N_0\right)}^{\mathcal{S}_C}$$

$$\Sigma = \Sigma_l + \Sigma_r = t_l G_l t_l^\dagger + t_r G_r t_r^\dagger \qquad \text{where} \quad \rho_l^\uparrow \neq \rho_l^\downarrow \quad \text{and} \quad \rho_r^\uparrow = \rho_r^\downarrow$$

Two possible strategies

„Mean-field“-expansion **„AES-like“-approach**

“Mean-field”-expansion:

$$\mathbf{M}(t) = \mathbf{M}_0 + \delta\mathbf{M}(t) \qquad V_d(t) = V_d^0 + \delta V_d(t)$$

$$i\mathcal{S}[\mathbf{M}, V_d] = \mathrm{tr}\ln\Big[-i\Big(\underbrace{G_{d,0}^{-1} + \frac{1}{2}\mathbf{M}_0\,\boldsymbol{\sigma} - V_d^0 - \Sigma}_{G_d^{-1}} + \delta\mathbf{M}\frac{\boldsymbol{\sigma}}{2} - \delta V_d\Big)\Big] + i\mathcal{S}_B + i\mathcal{S}_C$$

$$\text{expansion in } \delta\mathbf{M} \quad \longrightarrow \quad i\mathcal{S}_{\delta\mathbf{M}} = -\frac{1}{2}\mathrm{tr}\left[G_d\,\delta\mathbf{M}\frac{\sigma}{2}\,G_d\,\delta\mathbf{M}\frac{\sigma}{2}\right]$$

$$\text{expansion in } \delta V_d \quad \longrightarrow \quad i\mathcal{S}_{\delta V_d} = -\frac{1}{2}\mathrm{tr}\left[G_d\,\delta V_d\,G_d\,\delta V_d\right]$$

Determine the “auxiliary” Green's function by inverting:

$$G_d^{-1}(\epsilon) = \begin{pmatrix} 0 & \epsilon - \epsilon_\alpha + \mathbf{M}_0\frac{\boldsymbol{\sigma}}{2} - V_d^0 - i(\Gamma_l^\sigma + \Gamma_r) \\ \epsilon - \epsilon_\alpha + \mathbf{M}_0\frac{\boldsymbol{\sigma}}{2} - V_d^0 + i(\Gamma_l^\sigma + \Gamma_r) & -2i[\Gamma_l^\sigma F_l(\epsilon) + \Gamma_r F_r(\epsilon)] \end{pmatrix}$$

"auxiliary" Green's function:

$$G_d = \begin{pmatrix} G_d^K & G_d^R \\ G_d^A & 0 \end{pmatrix}$$

$$G_d^{R/A} = \frac{1}{\epsilon - \epsilon_\alpha + \mathbf{M}_0 \frac{\boldsymbol{\sigma}}{2} - V_d^0 \pm i(\Gamma_l^\sigma + \Gamma_r)}$$

relaxation rates (tunneling)

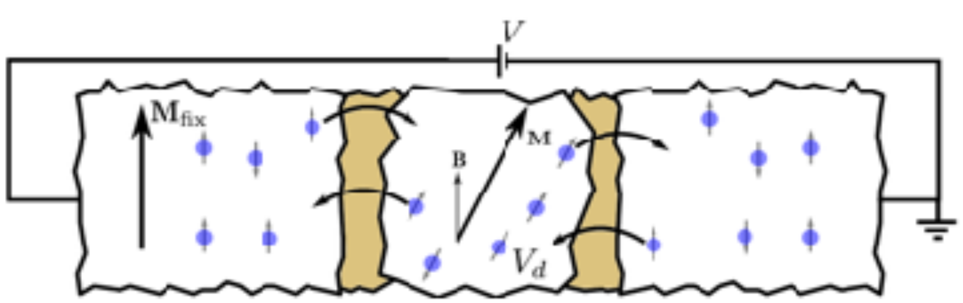
$$G_d^K = -2\pi i\ \delta_\Gamma(\epsilon - \epsilon_\alpha + \mathbf{M}_0 \frac{\boldsymbol{\sigma}}{2} - V_d^0)\ \frac{\Gamma_l^\sigma F_l(\epsilon) + \Gamma_r F_r(\epsilon)}{\Gamma_l^\sigma + \Gamma_r}$$

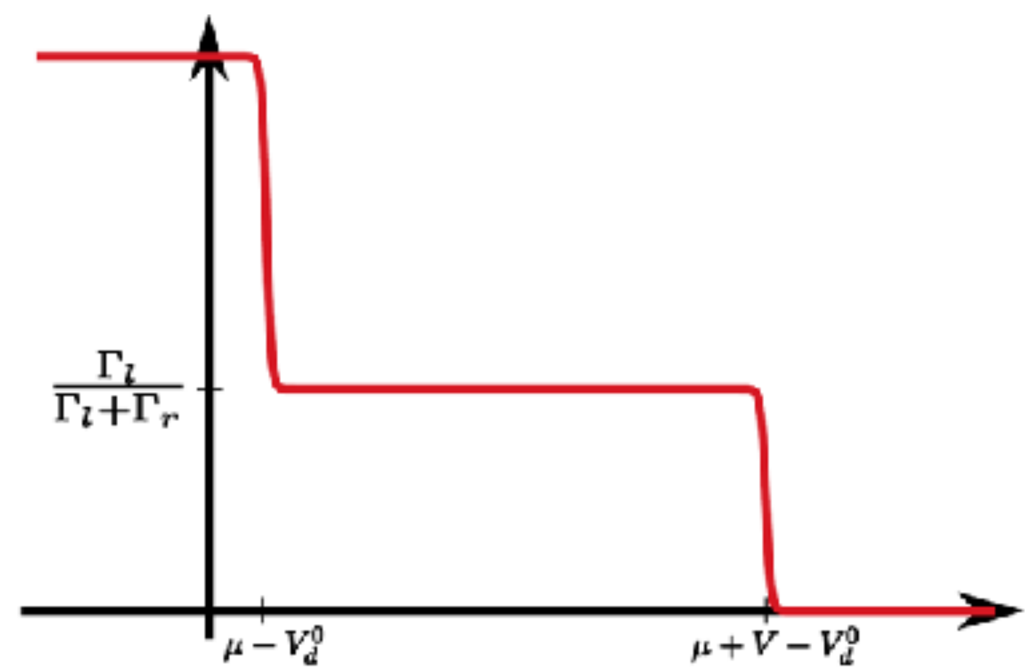
lead-distribution functions imposed upon the dot

Great strategy if $\delta\mathbf{M}$ and δV_d small

works also if $\mathbf{M_0(t)}$, $V_d^0(t)$ - slow

D. M. Basko, et al. PRB 79 064418 (2009)





AES-strategy:

$U(1)$ $SU(2)$

$$i\mathcal{S} = \mathrm{tr}\ln\left[-i\left(G_{d,0}^{-1} + \mathrm{M}\frac{\boldsymbol{\sigma}}{2} - V_d - \Sigma\right)\right] + i\mathcal{S}_B + i\mathcal{S}_C$$

(1) shift the time dependent fields to the tunneling part (gauge trafo):

$$U = \underbrace{\mathrm{e}^{-i\frac{\phi}{2}\sigma_z}\mathrm{e}^{-i\frac{\theta}{2}\sigma_y}\mathrm{e}^{i\frac{\phi-\chi}{2}\sigma_z}}_{SU(2)}\mathrm{e}^{-i\psi} \qquad \dot{\psi} = V_d$$

(2) expand in the self-energy (tunneling) (and in the Berry-phase):

$$i\mathcal{S} = \mathrm{tr}\ln\left[-i\left(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2}\sigma_z}_{G_{d,z}^{-1}} - Q_q - U^\dagger \Sigma U\right)\right] + i\mathcal{S}_B + i\mathcal{S}_C$$

AES-strategy:

$$i\mathcal{S} = \mathrm{tr}\ln\Big[-i\Big(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2}\sigma_z}_{G_{d,z}^{-1}} - Q_q - U^\dagger \Sigma U\Big)\Big] + i\mathcal{S}_B + i\mathcal{S}_C$$

$$i\mathcal{S}_{\mathrm{AES}} = -\mathrm{tr}\ \left[G_{d,z} U^\dagger \Sigma U\right] \qquad i\mathcal{S}_{\mathrm{WZNW}} = -\mathrm{tr}\ \left[G_{d,z} Q_q\right]$$

We know the self-energy

$$\Sigma_\sigma(\epsilon) \approx \begin{pmatrix} 0 & i(\Gamma_l^\sigma + \Gamma_r) \\ -i(\Gamma_l^\sigma + \Gamma_r) & -2i[\Gamma_l^\sigma F_l(\epsilon) + \Gamma_r F_r(\epsilon)] \end{pmatrix}$$

But what about the GF?

Which distribution should we use for the dot? (kinetic equation)

$$G_{d,z}^{-1}(\epsilon) = \begin{pmatrix} 0 & \epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z - i0 \\ \epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z + i0 & -2i0\, F_?(\epsilon) \end{pmatrix}$$

“improved” AES-like-strategy:

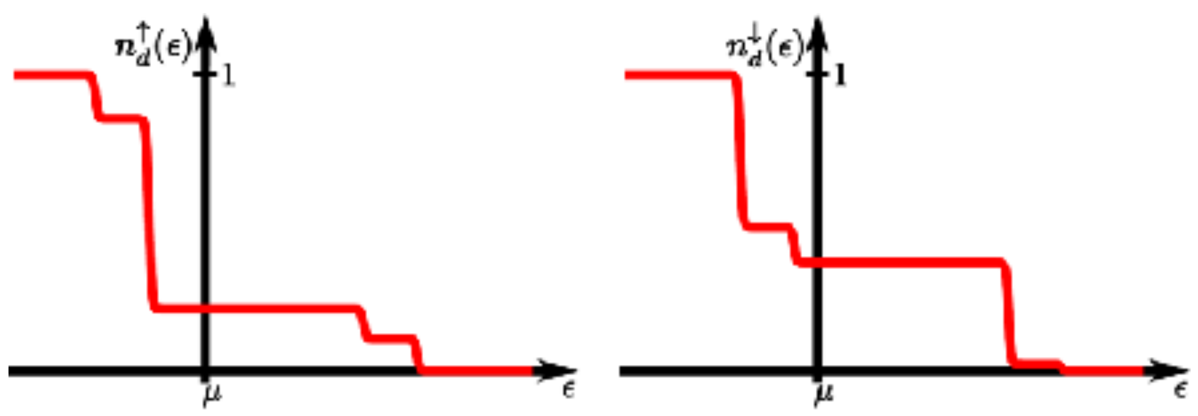
expand around a classical saddle point (persistent precessions)

$$i\mathcal{S} = \mathrm{tr}\ln\Big[-i\Big(\underbrace{G_{d,0}^{-1} + \frac{M_0}{2}\sigma_z - U_0^\dagger \Sigma U_0}_{G_d^{-1}} - Q_q - (U^\dagger\Sigma U - U_0^\dagger\Sigma U_0)\Big)\Big] + i\mathcal{S}_B + i\mathcal{S}_C$$

$$i\mathcal{S}_{\mathrm{AES}} = -\mathrm{tr}\left[G_d(U^\dagger\Sigma U - U_0^\dagger\Sigma U_0)\right]$$

$$G_d(\epsilon) = \begin{pmatrix} -2\pi i\ \delta_{\Gamma_\sigma(\theta_0)}(\epsilon - \epsilon_\alpha + \frac{M_0}{2}\sigma_z)\ F_d^\sigma(\epsilon) & \frac{1}{\epsilon-\epsilon_\alpha+\frac{M_0}{2}\sigma_z+i\Gamma_\sigma(\theta_0)} \\ \frac{1}{\epsilon-\epsilon_\alpha+\frac{M_0}{2}\sigma_z-i\Gamma_\sigma(\theta_0)} & 0 \end{pmatrix}$$

$$\begin{aligned} F_d^\sigma(\epsilon) = \frac{1}{\Gamma_\sigma(\theta_0)}\Big[&\cos^2\frac{\theta_0}{2}\Gamma_l^\sigma F(\epsilon - \sigma B_- + V_d^0 - V) \\ &+ \sin^2\frac{\theta_0}{2}\Gamma_l^{\bar\sigma} F(\epsilon - \bar\sigma B_+ + V_d^0 - V) \\ &+ \cos^2\frac{\theta_0}{2}\Gamma_r F(\epsilon - \sigma B_- + V_d^0) \\ &+ \sin^2\frac{\theta_0}{2}\Gamma_r F(\epsilon - \bar\sigma B_+ + V_d^0)\Big] \end{aligned}$$



$$B_\pm \equiv B_0(1 \pm \cos\theta_0)/2$$

Landau-Lifshitz-Gilbert-Slonczewski equation:

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = -\mathbf{B}\times\mathbf{M} - \alpha(\theta)\frac{\mathbf{M}}{M_0}\times\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} + \mathbf{M}\times\left(\frac{\mathbf{I}_s}{S}\times\frac{\mathbf{M}}{M_0}\right)$$

Gilbert-damping:

$$\alpha(\theta) = \frac{\tilde{g}_l(\theta)+\tilde{g}_r}{S}$$

Kirchhoff's law:

$$C\dot{V}_d = I_l - I_r$$

EOM

$$I_s = g_s(V - V_d) + \Delta I_s(\theta_0)$$

$$I_r = 4g_r V_d + \Delta I_r(\theta,\theta_0)$$

$$I_l = 4g(\theta)(V - V_d) - g_s \sin^2\theta\,\dot{\phi} + \Delta I_l(\theta,\theta_0)$$

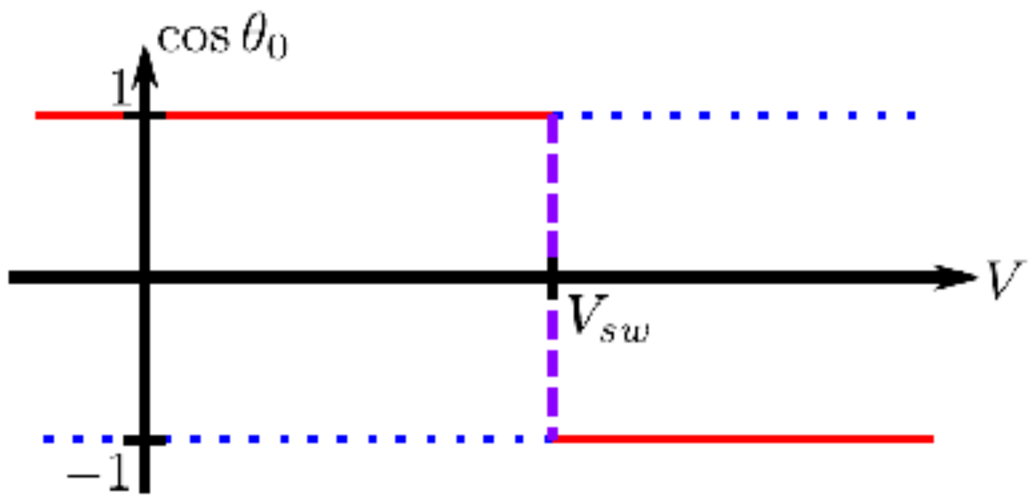
$$\Delta I_s(\theta_0) = g_s(V_d^0 - V) + S\,\alpha(\theta_0)\,\frac{2\Gamma_\Delta\Gamma_r V - \Gamma_\Delta^2\sin^2\theta_0\,B_0}{\Gamma_\Sigma^2 - \cos^2\theta_0\Gamma_\Delta^2}$$

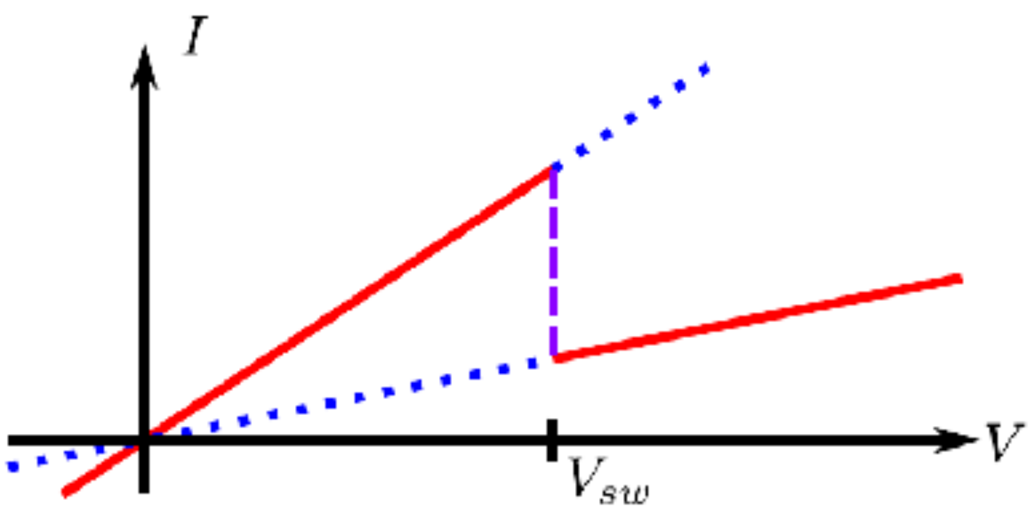
$$\Delta I_l(\theta,\theta_0) = 4g_l(V_d^0 - V) + \left[\frac{\cos^2\frac{\theta}{2}\,g_l^{\uparrow\uparrow} + \sin^2\frac{\theta}{2}\,g_l^{\uparrow\downarrow}}{\Gamma_\uparrow(\theta_0)} + \frac{\cos^2\frac{\theta}{2}\,g_l^{\downarrow\downarrow} + \sin^2\frac{\theta}{2}\,g_l^{\downarrow\uparrow}}{\Gamma_\downarrow(\theta_0)}\right]\left(\Gamma_r V - \Gamma_\Delta\sin^2\theta_0\,\frac{B_0}{2}\right)$$

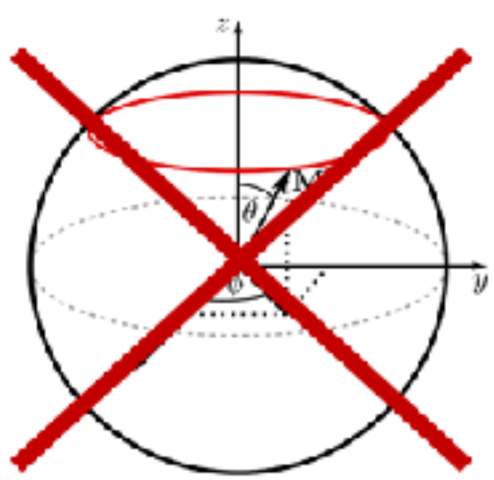
$$\Delta I_r(\theta,\theta_0) = 4g_r(V - V_d^0) - \left[\frac{\cos^2\frac{\theta}{2}\,g_r^{\uparrow\uparrow} + \sin^2\frac{\theta}{2}\,g_r^{\uparrow\downarrow}}{\Gamma_\uparrow(\theta_0)} + \frac{\cos^2\frac{\theta}{2}\,g_r^{\downarrow\downarrow} + \sin^2\frac{\theta}{2}\,g_r^{\downarrow\uparrow}}{\Gamma_\downarrow(\theta_0)}\right]\left(\Gamma_r V - \Gamma_\Delta\sin^2\theta_0\,\frac{B_0}{2}\right)$$

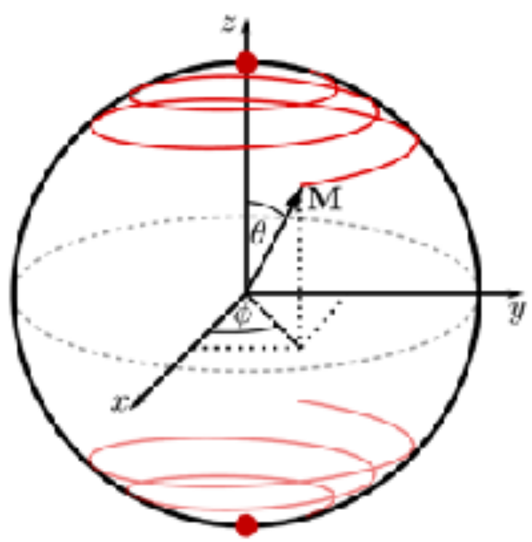
Stationary solutions:

$$\dot{\theta} = -\frac{\sin\theta}{1+\alpha^2}\left[\alpha(\theta)B + I_s(\theta_0)/S\right] \qquad I_l = 4g(\theta)(V - V_d) - g_s \sin^2\theta\dot{\phi} + \Delta I_l(\theta,\theta_0)$$



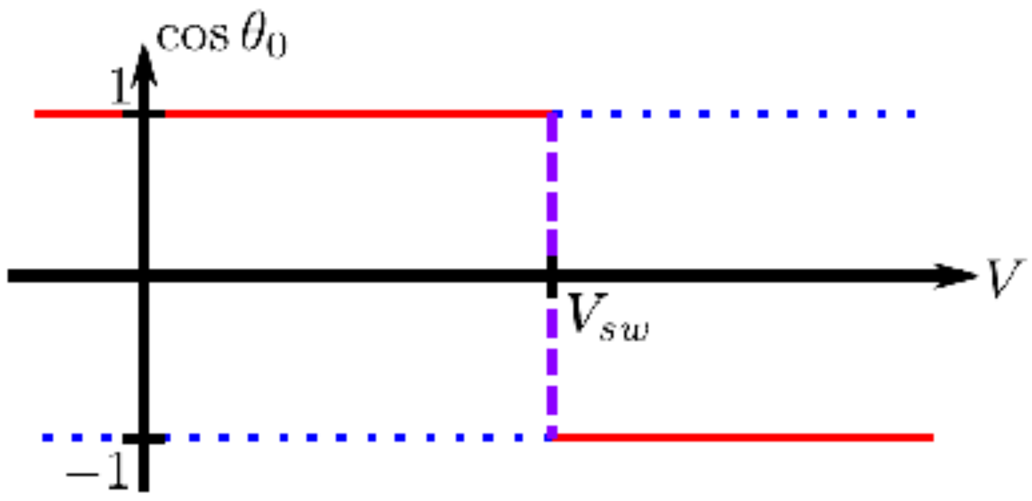


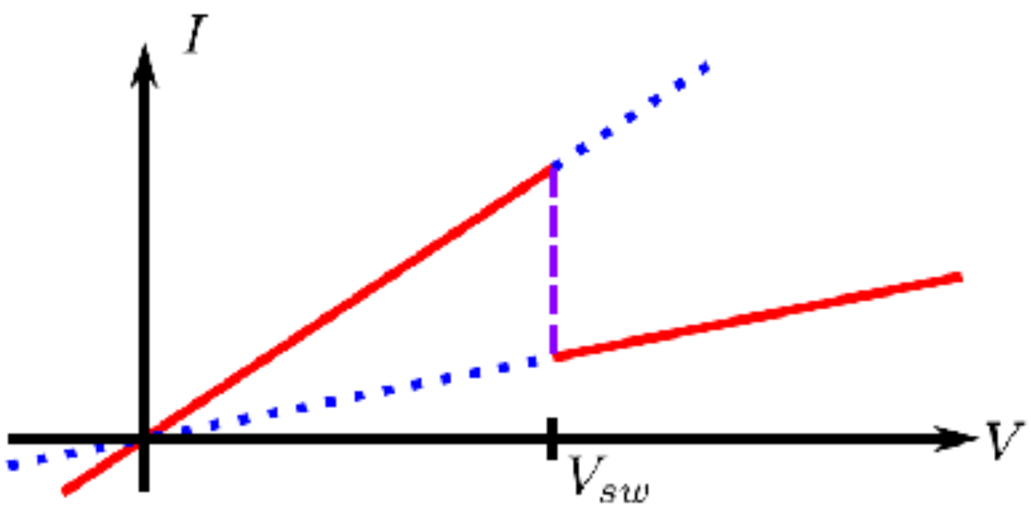




Stationary solutions:

$$\dot{\theta} = -\frac{\sin\theta}{1+\alpha^2}\left[\alpha(\theta)B + I_s(\theta_0)/S\right] \qquad I_l = 4g(\theta)(V - V_d) - g_s \sin^2\theta\,\dot{\phi} + \Delta I_l(\theta,\theta_0)$$





Relaxation dynamics:

$$\delta\dot{\theta} = -\cos\theta_0\,\alpha(\theta_0)B\left[1 - \frac{V}{V_{sw}}\right]\delta\theta$$

Critical slowing down!

$$C\delta\dot{V}_d = -4(g_l + g_r)\delta V_d$$

Where did the stationary potential go? Zero modes!