Kondo Impurities in Helical Luttinger Liquids

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Outline

I. Introduction, model
(helical electrons, Kondo impurities),
various regimes

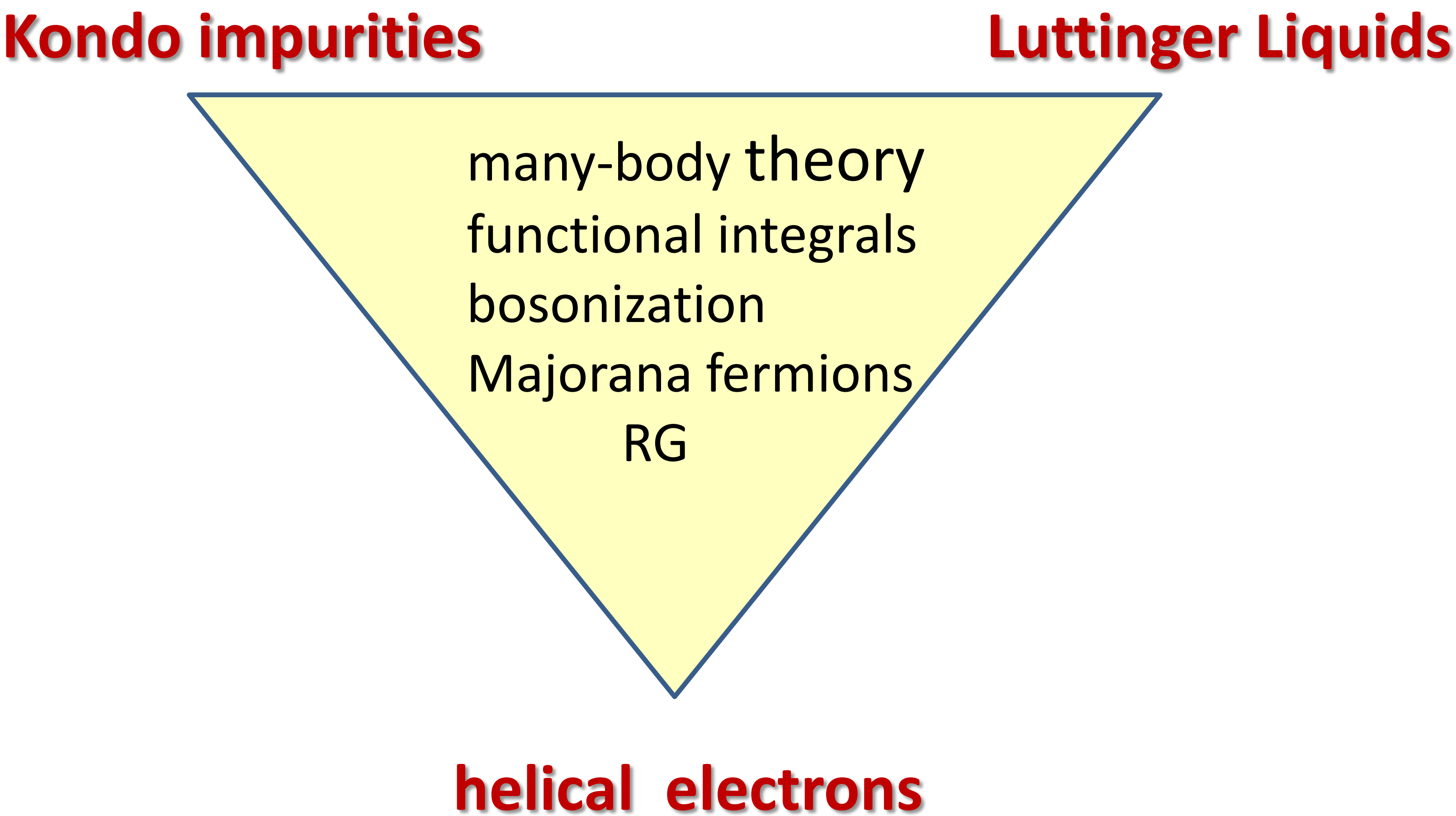
II. Exploration of the model and basic results
1. Effective spin-spin (RKKY) interaction
2. Derivation of a slow effective action

III. Electron transport (conductivity)

IV. Effects of e-e interactions, Luttinger liquid

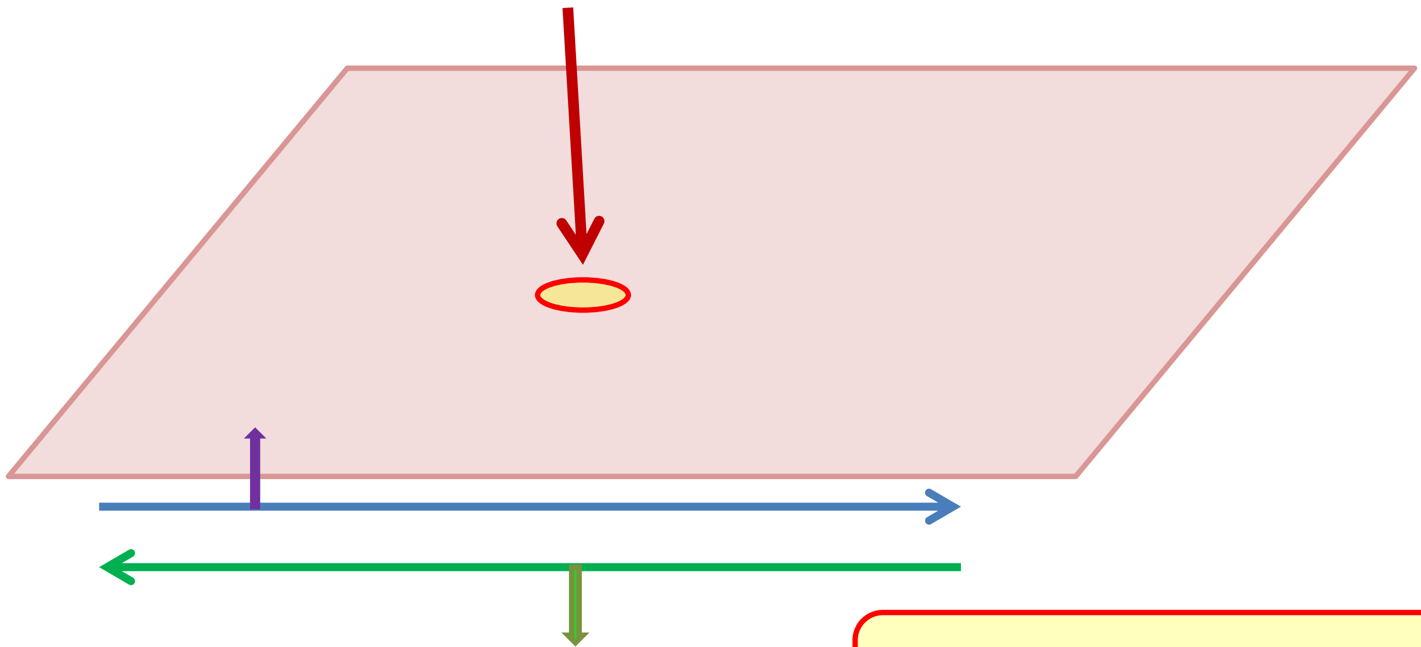
V. RKKY *vs* Kondo in Helical Luttinger liquid

Summary and open questions


Kondo impurities
Luttinger Liquids
many-body theory
functional integrals
bosonization
Majorana fermions
RG
helical electrons

Basic properties of a generic 2D Topological Insulator:

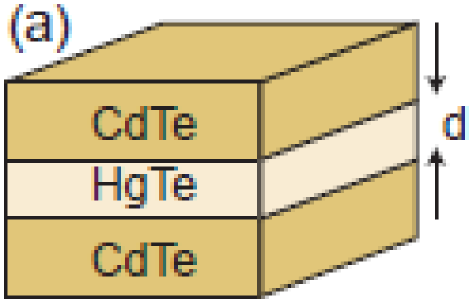
2D bulk - insulator: bulk electron spectrum is gapped, impurity states are localized

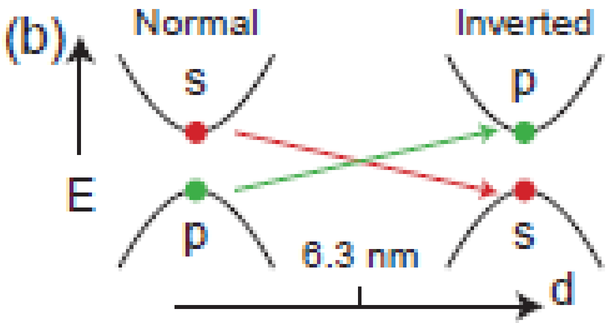


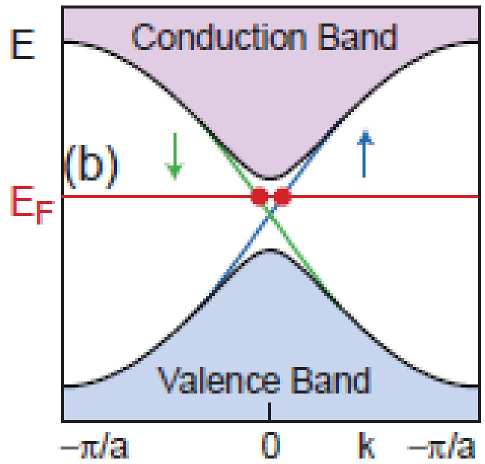
Edge Modes are Helical

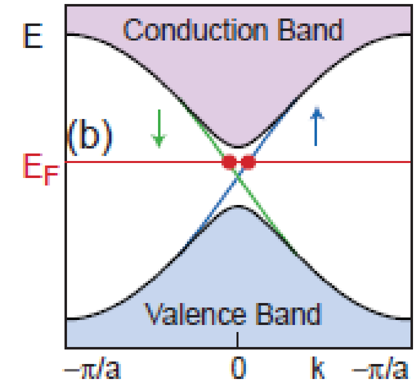
Realization of helical edge modes: 2D Topological Insulator

Kane and Mele (2005);
Bernevig, T. L. Hughes, and S. C. Zhang (2006)
(CdTe-HgTe-CdTe)

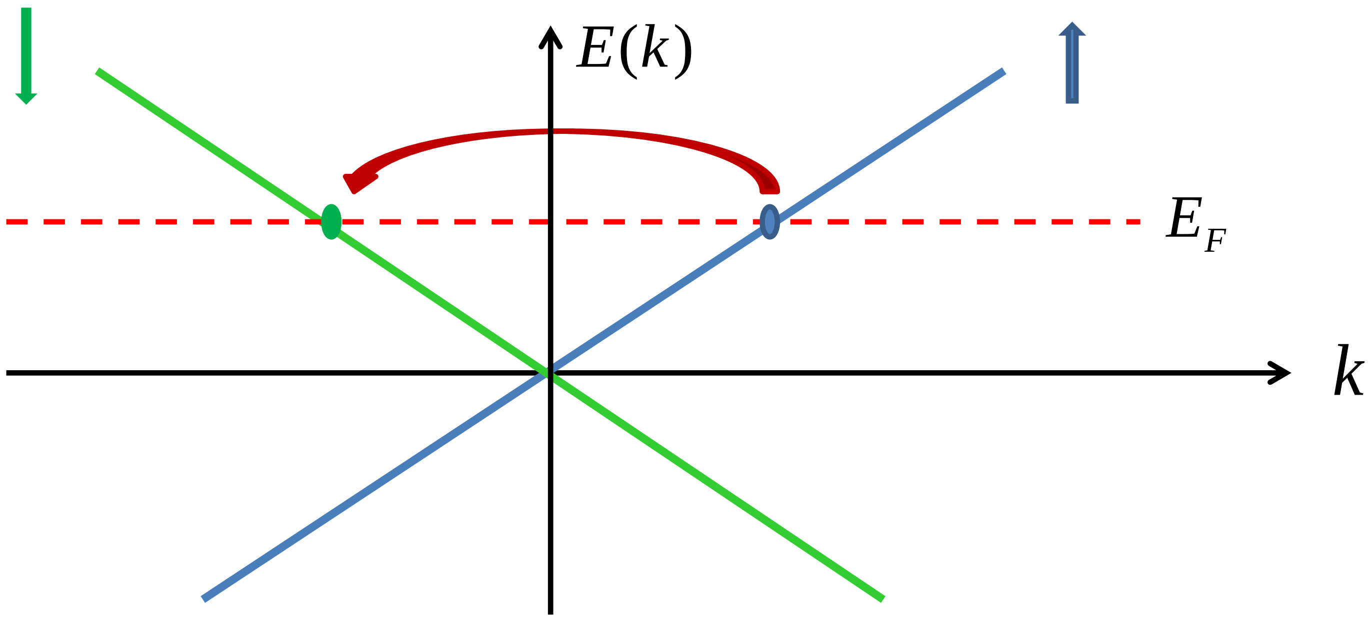








Elastic backscattering?



No Backscattering by a *Potential Disorder*

No Anderson Localization

An Ideal Conductor?
A new state of matter?

Is this 1D system protected from Anderson Localization at $T \to 0$?

Images of Edge Current in InAs/GaSb Quantum Wells

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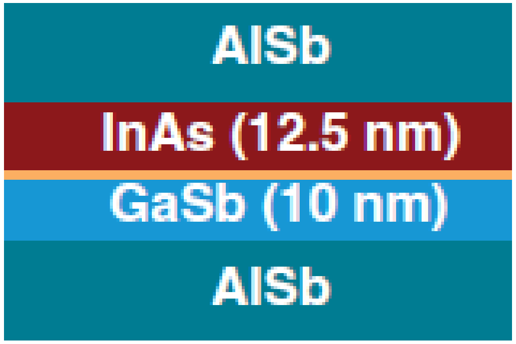
Quantum spin Hall devices with edges much longer than several microns do not display ballistic transport; that is, their measured conductances are much less than e^2/h per edge. We imaged edge currents in InAs/GaSb quantum wells with long edges and determined an effective edge resistance. Surprisingly, although the effective edge resistance is much greater than h/e^2, it is independent of temperature up to 30 K within experimental resolution. Known candidate scattering mechanisms do not explain our observation of an effective edge resistance that is large yet temperature independent.



What is the nature of the resistance?

To be backscattered helical electrons should change spin

⟹ Spin (Kondo) “impurities” may play a role!

(motivation of the lecture topic)

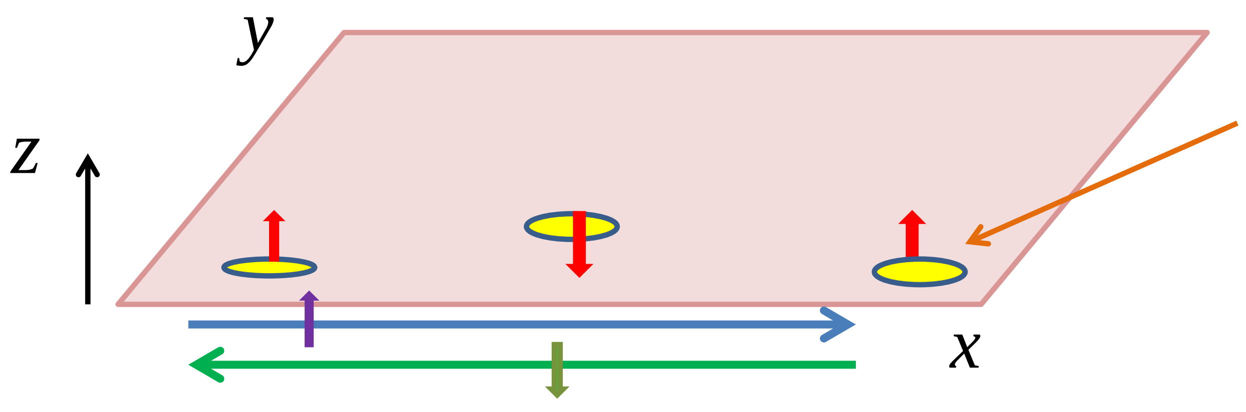
Outline:

Dense impurity ensemble

Single impurity

Pairs of impurities in HLL

Spin (Kondo) impurities located near the helical edge

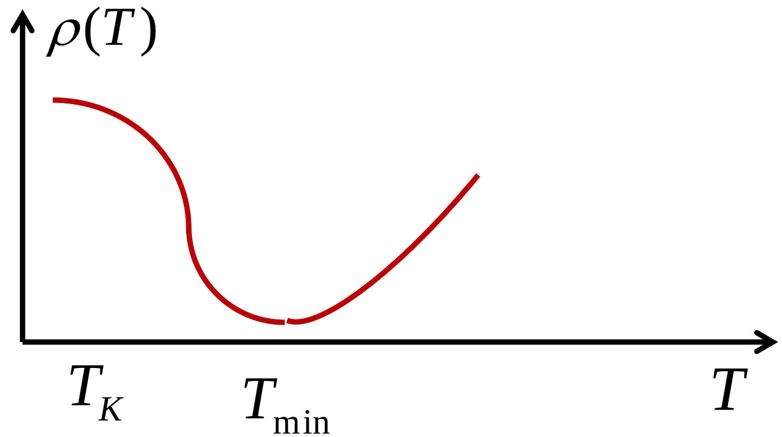


A single-electron "impurity" (due to Coulomb blockade)

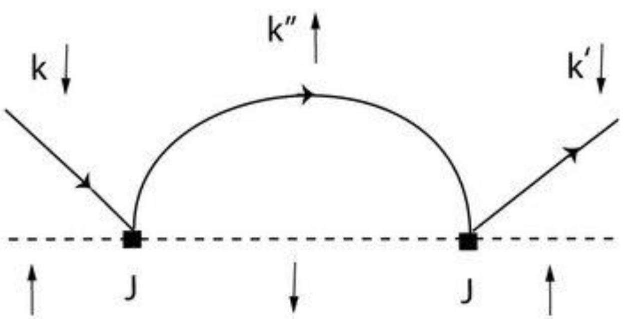
Single impurity physics: Kondo effect in bulk conductors

Jun Kondo *"Resistance Minimum in Dilute Magnetic Alloys".*
Progress of Theoretical Physics. ***32****: 37*(1964)

$$\rho(T) = a\frac{T^2}{E_F} + bT^5 + + c\log\left(\frac{E_F}{T}\right)$$



$$H = J\sum_{\vec{k},\vec{k}'} \vec{S}\, c^{+}_{\vec{k}'\alpha}\vec{\sigma}_{\alpha\beta}c_{k\beta}$$



A “bound state at $T < T_K$

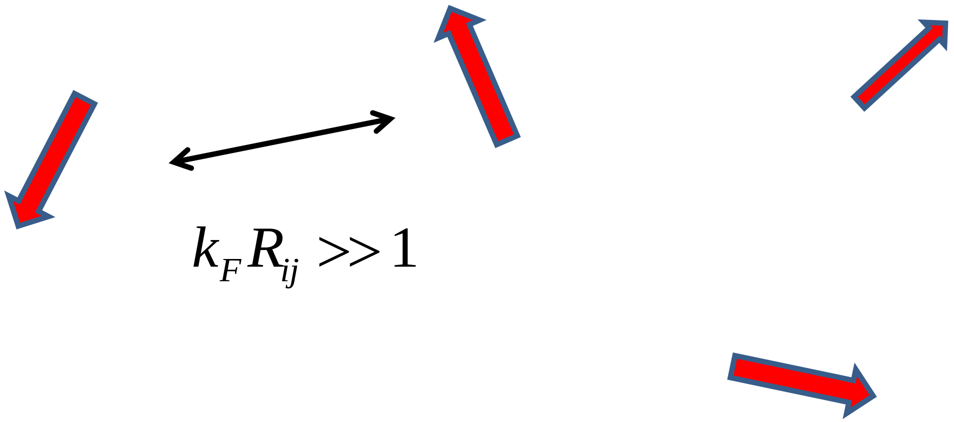
$$T_K \sim E_F \exp\left(-\frac{1}{\rho(E_F)J}\right)$$

Conclusion:

a very low density ensemble of Kondo impurities does not cause backscattering at $T \to 0$

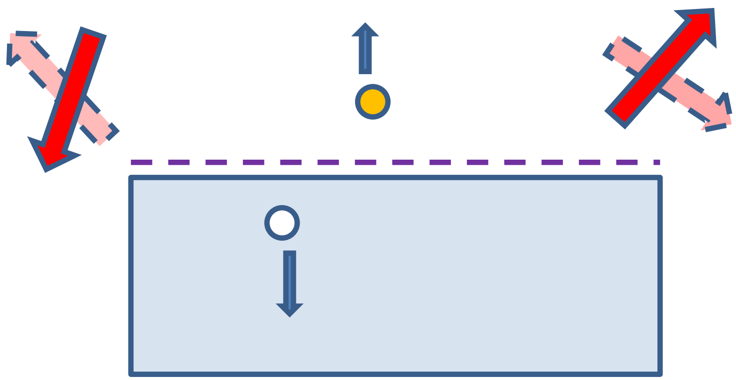
A (moderately) dense ensemble of spin impurities:

(Bulk conductor, non-interacting electrons)



$$k_F R_{ij} >> 1$$

RKKY (*Ruderman–Kittel–Kasuya–Yosida, 1954-1957)*
coupling mechanism through conduction electrons



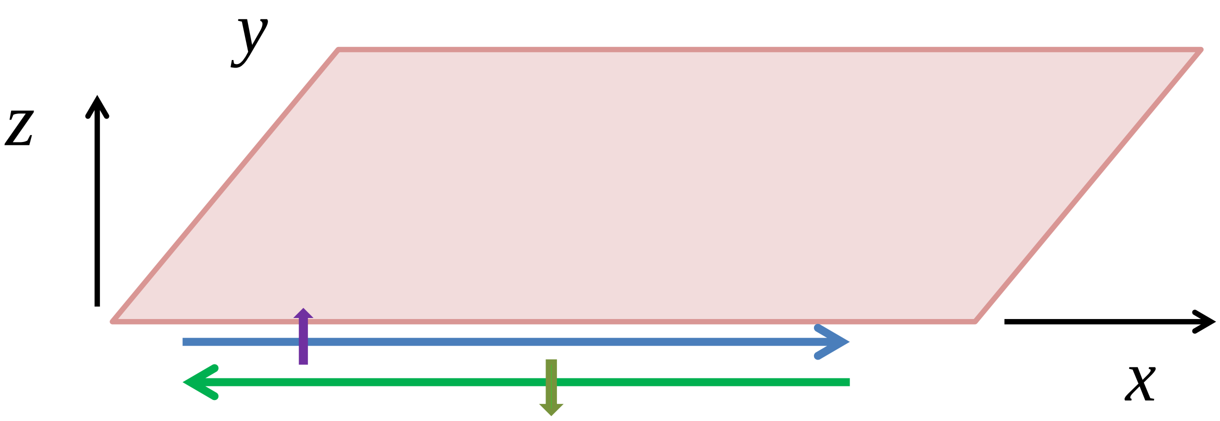
$$H_{\text{int}} = J\sum_{j}\sum_{\vec{k},\vec{k}'}\vec{S}\; c^{+}_{\vec{k}\alpha}\vec{\sigma}_{\alpha\beta}c_{k'\beta}\; e^{i(k-k')r_j}$$

$$H_{eff} = -\sum_{k\alpha,k'\beta}\frac{< s_1^{'}, s_2^{'}; vac \,|\, H_{\text{int}} \,|\, s_1^{'}, s_2; k\alpha, k'\beta >< s_1^{'}, s_2; k\alpha, k'\beta \,|\, H_{\text{int}} \,|\, s_1, s_2; vac >}{\varepsilon_k - \varepsilon_{k'}}$$

$$H_{eff}^{RKKY} \propto J^2 \sum_{j\neq l}\frac{\vec{S}_j\vec{S}_l\cos(2k_F R_{jl})}{R_{jl}^d}$$

RKKY dominates over Kondo,
if $E_{RKKY} >> T_K$

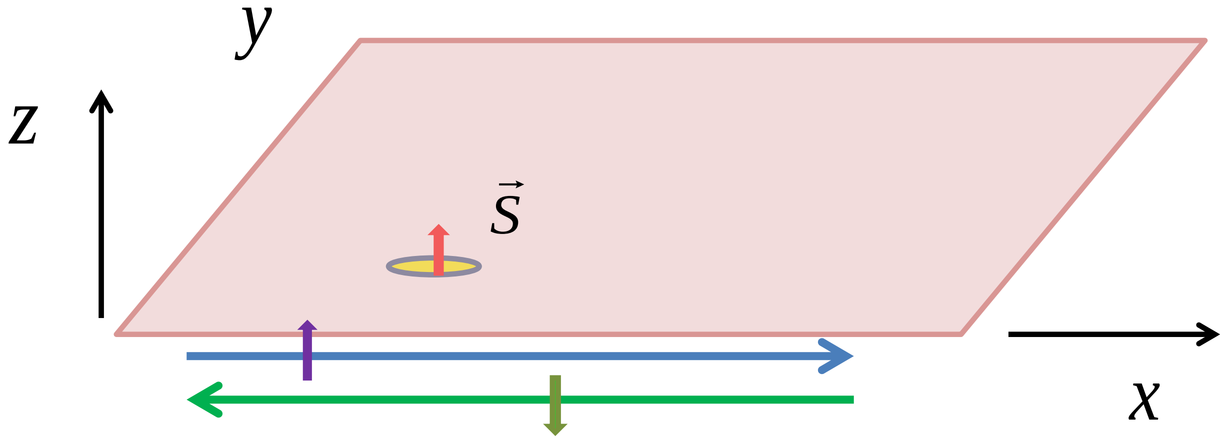
Noninteracting helical electrons



Hamiltonian: $H = H_e + H_{e-S}$

Free helical electrons: $H_e = -iv_F \int dx \left[\Psi^+_{R(\uparrow)}(x) \partial_x \Psi_{R(\uparrow)} - \Psi^+_{L(\downarrow)}(x) \partial_x \Psi_{L(\downarrow)} \right]$

Electron operator: $\Psi(x) = \Psi_{R\uparrow}(x) e^{ik_F x} + \Psi_{L\downarrow}(x) e^{-ik_F x}$



Electron-spin interaction

$$J_z\sigma_z S^z + J_x\sigma_x S^x + J_y\sigma_y S^y = \underbrace{J_z\sigma_z S^z + \frac{J_\parallel}{2}(\sigma_+ S^- + \sigma_- S^+)}_{S^z\text{- conservation}} + \frac{\delta J}{2}\underbrace{(\sigma_+ S^+ + \sigma_- S^-)}_{\text{no } S^z\text{- conservation}}$$

$$J_\parallel = \frac{1}{2}(J_x + J_y), \quad \delta J = \frac{1}{2}(J_x - J_y)$$

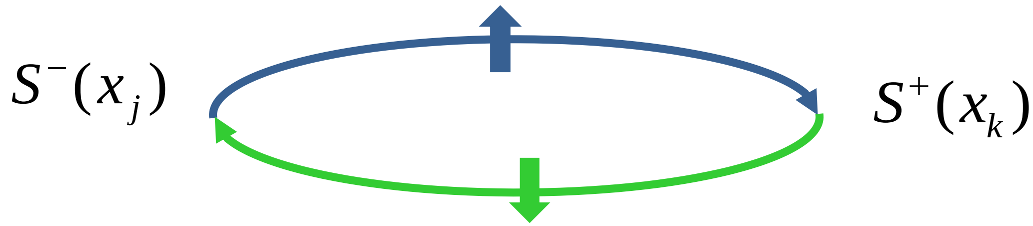
(weak anisotropy: $|\delta J| \ll |J_\parallel|$)

$$H_{e-S} = \sum_j \left[J_z S^z \left(\Psi^+_{R(\uparrow)}\Psi_{R(\uparrow)} - \Psi^+_{L(\downarrow)}\Psi_{L(\downarrow)} \right) + \frac{J_\parallel}{2}\left(S_j^+ \Psi^+_{L(\downarrow)} \mathrm{e}^{2ik_F x_j}\, \Psi_{R(\uparrow)} + S_j^- \Psi^+_{R(\uparrow)} \mathrm{e}^{-2ik_F x_j}\, \Psi_{L(\downarrow)} \right) \right] \quad U(1)$$

$$+ \sum_j \frac{\delta J}{2}\left(S_j^+ \Psi^+_{R(\uparrow)} \mathrm{e}^{-2ik_F x_j}\, \Psi_{L(\downarrow)} + S_j^- \Psi^+_{L(\downarrow)} \mathrm{e}^{2ik_F x_j}\, \Psi_{R(\uparrow)} \right) \quad \cancel{U}(1)$$

Effective spin-spin ("RKKY") interaction

(Simplifications: no e-e interactions; unbroken $U(1)$ symmetry)



Helical electrons

$$H_{S-S} = -\frac{J_\parallel^2}{8\pi v_F}\sum_{j\neq k}\frac{S_j^+ S_k^- e^{2ik_F(x_j - x_k)} + h.c.}{|x_j - x_k|}$$

"Usual" electrons

$$H_{S-S} \propto \frac{J^2}{v_F}\sum_{j\neq k}\frac{\vec{S}_j \vec{S}_k \cos[2k_F(x_j - x_k)]}{|x_j - x_k|}$$

Features of helicity:

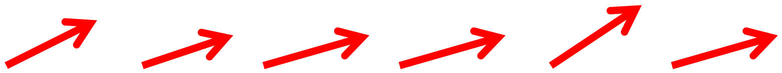
1. No $S_j^z S_k^z$ interaction;
2. Factors $e^{2ik_F(x_j - x_j)}$ instead of $\cos[2k_F(x_j - x_j)]$

Can be removed by $S_j^\pm \to S_j^\pm e^{\mp 2ik_F x_j}$

$U(1)$

For rotated spins: $$H_{S-S}^{(eff)} = -\frac{J_{\parallel}^2}{8\pi v_F}\sum_{j\neq k}\frac{S_j^+ S_k^- + h.c.}{|x_j - x_k|}$$

A tendency to form a macroscopically ordered in-plane state



(suppresses Kondo effect!)

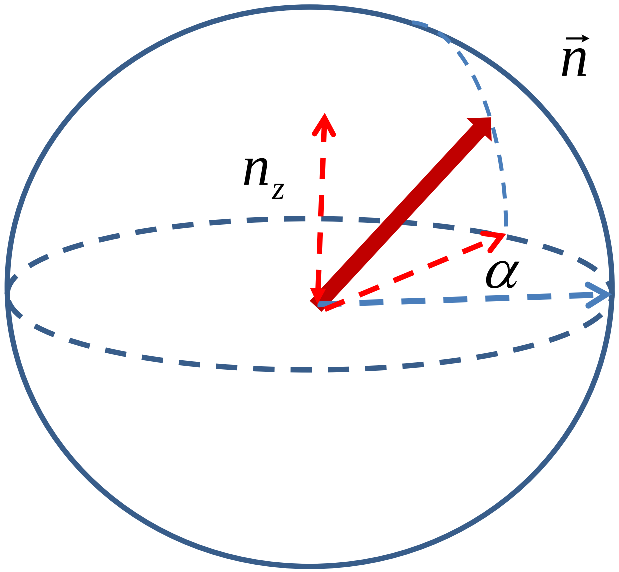
A possibility of “macroscopic” description:

Coherent representation for spins:

$$\vec{S}_j \to S\vec{n}_j\,; \quad |\vec{n}_j| = 1; \quad S = \frac{1}{2}$$

$$\Downarrow$$

$$S_j^z \to \frac{1}{2} n_{z,j}\,; \quad S_j^{\pm} \to \frac{1}{2}\sqrt{1 - n_{z,j}^2}\, \mathrm{e}^{\pm i\alpha_j}$$



Effective spin-spin interaction in the coherent spin representation:

(for rotated spins!)

$$H_{S-S}^{(eff)} = -\frac{J_{\parallel}^2 S^2}{4\pi v_F} \sum_{j \neq k} \frac{\sqrt{(1-n_{z,j}^2)(1-n_{z,k}^2)}\cos(\alpha_j - \alpha_k)}{|x_j - x_k|}$$

"Classical" ground state: $n_{z,j} = 0, \quad \alpha_j = Const$

Task: *to account for quantum fluctuations*

Low energy -- long-range limit ⟹ continual description:

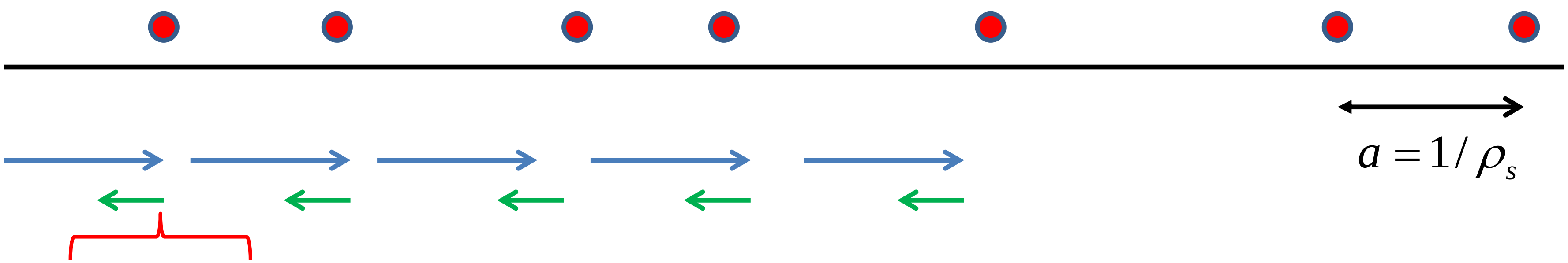
$$n_{z,j},\ \alpha_j \to n_z(x), \alpha(x)|_{x=x_j}\ ; \quad \sum_j \ldots \to \int dx\, \rho_S(x) \ldots$$

Linear density of Kondo impurities: $\rho_s = 1/a$

Model parameters and “simple hand-waving physics”

Weak electron–spin interaction: $\lambda_{Kondo} = \rho(E_F)J \sim \frac{J_{\parallel}}{v_F} << 1$

Irrelevance of Kondo effect: $T_{Kondo} \sim E_F \exp\left(-\frac{1}{\lambda_{Kondo}}\right) << E_{RKKY} \sim \frac{J_{\parallel}^2}{v_F a}$



Reflection amplitude: $\sim J/v_F << 1$

Reflection length: $\xi \sim a v_F / J_{\parallel} \equiv v_F/\Delta; \quad \Delta \sim J_{\parallel}/a$

$\xi >> a$ ⟹ continual description is justified

Statistical sum in functional integral representation

$$Z = \int D\overline{\Psi} D\Psi D\vec{S} \exp[-S(\overline{\Psi}, \Psi; \vec{S})]$$

Matsubara action in the coherent representation for spins:

$$\mathsf{S}(\overline{\Psi}, \Psi; \vec{S}) = \int dx d\tau \, \overline{\Psi} \partial_\tau \Psi + \mathsf{S}_S + \int dx d\tau \, H(\overline{\Psi}, \Psi; \vec{S})$$

Phase ("Berry") action:

$$\mathsf{S}_S = iS \sum_j \int_0^\beta d\tau \left[1 - n_{z,j}(\tau)\right] \partial_\tau \alpha_j(\tau)$$

(An analog of $i\int p(t)\dot{x}(t)dt$ in the usual path integral)

Integration measure: $D\vec{S} = \prod_j Dn_{z,j}(\tau) D\alpha_j(\tau)$

Matsubara Action:

$$S[\Psi,\vec{S}] = S_{e-S}[\Psi,\vec{S}] + S_S[\vec{S}]$$

Electron-spin part:

$$S_{e-S}[\Psi,\vec{S}] = \int d\tau dx \left(\overline{\Psi}_{R(\uparrow)}, \overline{\Psi}_{L(\downarrow)}\right) \begin{bmatrix} \partial_+, & \Delta\sqrt{1-n_z^2}[e^{-i\alpha} + \eta(x)e^{i\alpha}] \\ \Delta\sqrt{1-n_z^2}[e^{i\alpha} + \eta^*(x)e^{-i\alpha}], & \partial_- \end{bmatrix} \begin{pmatrix} \Psi_{R(\uparrow)} \\ \Psi_{L(\downarrow)} \end{pmatrix}$$

here: $\partial_\pm = \partial_\tau \mp i v_F \partial_x; \quad \Delta = \dfrac{\rho_s J_\parallel}{4}$ (For simplicity: $J_z \to 0$)

Spin ("Berry") action: $S_S[n_z,\alpha] = -iS \int d\tau dx\, \rho_S(x) n_z(x,\tau) \partial_\tau \alpha(x,\tau)$

$\eta(x) \propto \dfrac{\delta J}{J_\parallel} e^{4ik_F x}$ - *random function:* $\langle \eta^*(x)\eta(x')\rangle = d\, a\delta(x-x'); \;\; d \ll 1$

If $\alpha(x,\tau) = Const$, *electrons are gapped!*

$E(p) = \pm\sqrt{v_F^2 p^2 + \Delta^2}$

Gauge transformation (to account for variations of $\alpha(x,\tau)$):

$$\Psi_{R(\uparrow)} = e^{-i\frac{\alpha}{2}} \psi_{R(\uparrow)}, \quad \Psi_{L(\downarrow)} = e^{i\frac{\alpha}{2}} \psi_{L(\downarrow)}$$

$$\overline{\Psi}_{R(\uparrow)} = e^{i\frac{\alpha}{2}} \overline{\psi}_{R(\uparrow)}, \quad \overline{\Psi}_{L(\downarrow)} = e^{-i\frac{\alpha}{2}} \overline{\psi}_{L(\downarrow)}$$

After gauge transformation

$$S_{e-S}[\Psi,\vec{S}] \to S_{e-S}[\psi,n_z,\alpha] + S_{an}[\alpha]$$

$$S_{e-S}[\psi,n_z,\alpha] = \int d\tau dx \left(\overline{\psi}_{R(\uparrow)},\overline{\psi}_{L(\downarrow)}\right) \begin{bmatrix} \partial_+ - \frac{i}{2}\partial_+\alpha, & \Delta\sqrt{1-n_z^2}(1+\eta e^{2i\alpha}) \\ \Delta\sqrt{1-n_z^2}(1+\eta e^{-2i\alpha}), & \partial_- + \frac{i}{2}\partial_+\alpha \end{bmatrix} \begin{pmatrix} \psi_{R(\uparrow)} \\ \psi_{L(\downarrow)} \end{pmatrix}$$

$$S_{an}[\alpha] = \frac{v_F}{8\pi}\int d\tau dx (\partial_x \alpha)^2$$

chiral anomaly (contribution of Jacobian)

Electron density in new variables:

$$\rho_{el} = \overline{\Psi}_{R(\uparrow)}\Psi_{R(\uparrow)} + \overline{\Psi}_{L(\downarrow)}\Psi_{L(\downarrow)} \to \overline{\psi}_{R(\uparrow)}\psi_{R(\uparrow)} + \overline{\psi}_{L(\downarrow)}\psi_{L(\downarrow)} - \frac{\partial_x\alpha}{2\pi}$$

Physical properties of various fields:

ψ - gapped fermions, $E(p) = \pm\sqrt{v_F^2 p^2 + \Delta^2}$, “correlation length” $\xi_0 = \frac{v_F}{\Delta_0}$

α - massless (“Goldstone”) bosons (spin rotations);

n_z - massive (“non-dynamical”) boson field → can be integrated out

Collecting things together ($U(1)$):

$$\mathsf{S} \to \mathsf{S}[\psi,\alpha] = \mathsf{S}_{e-S}[\psi,\alpha] + \mathsf{S}[\alpha]$$

Gapped fermions perturbed by boson field

Free boson field

$$\mathsf{S}_{e-S}[\psi,\alpha] = \int d\tau dx \left(\overline{\psi}_{R(\uparrow)}, \overline{\psi}_{L(\downarrow)}\right) \begin{bmatrix} \partial_+ - \frac{i}{2}\partial_+\alpha, & \Delta_0 \\ \Delta_0, & \partial_- + \frac{i}{2}\partial_+\alpha \end{bmatrix} \begin{pmatrix} \psi_{R(\uparrow)} \\ \psi_{L(\downarrow)} \end{pmatrix}$$

Free boson field:

$$\mathsf{S}[\alpha] = \int d\tau dx \frac{v_F}{8\pi u_0^2}[(\partial_\tau\alpha)^2 + u_0^2(\partial_x\alpha)^2]$$

$$u_0 = \frac{J_\parallel}{2\pi}\left(\ln\frac{E_B}{\Delta_0}\right)^{1/2} << \; v_F \qquad \Delta_0 = \frac{\rho_S J_\parallel}{4}$$

Range of applicability:

$$a << \xi_0 = \frac{v_F}{\Delta_0} \sim a\frac{v_F}{J_\parallel} \to \quad J_\parallel << v_F \quad \leftrightarrow \quad K_\alpha^{(0)} \equiv \frac{4u_0}{v_F} << 1$$

I. No anisotropy, no disorder ($\Delta = \Delta_0 = \frac{\rho_S J_\parallel}{4} = Const$):

Electron density correlation:

$$\langle \rho_{el}(-q,-\omega)\rho_{el}(q,\omega)\rangle = \frac{v_F q^2}{2\pi}\left[\frac{1}{6\Delta_0^2} + \frac{u_0^2}{v_F^2}\frac{1}{\omega^2 + u_0^2 q^2}\right]$$

Contribution of gapped fermions

Contribution of "free" spin action: $\sim hL^{(0)}h \quad ; \quad L^{(0)}(q,\omega) = 1/(\omega^2 + u^2 q^2)$

$$S[\alpha;h] = \int d\tau dx \left[\frac{v_F}{8\pi u^2}[(\partial_\tau \alpha)^2 + u^2(\partial_x \alpha)^2] - \frac{1}{2\pi} h\partial_x \alpha\right]$$

Electron current correlation:

$$\langle j_{el}(-q,-\omega) j_{el}(q,\omega)\rangle_{q,\omega\to 0} = -\frac{e^2\omega^2}{q^2}\langle \rho_{el}(-q,-\omega)\rho_{el}(q,\omega)\rangle \sim -\frac{e^2}{2\pi}\frac{u_0^2}{v_F^2}\frac{\omega^2}{\omega^2 + u_0^2 q^2}$$

Conductivity:

$$\sigma(\Omega) = \frac{\langle j_{el}(-q,-\omega) j_{el}(q,\omega)\rangle_{q=0, i\omega\to\Omega}}{i\Omega} \propto \frac{1}{-i\Omega} \Longleftrightarrow \text{ballistic}$$

$$\mathrm{Re}\,\sigma(\Omega) = 0$$

II. No anisotropy, weak static disorder:

$$u_0 \to u(x) = u_0[1+\mu(x)/2]$$

$$<\mu(x)\mu(x')>= \mu_0^2 a\delta(x-x'); \quad \mu_0^2 << 1$$

$$S[h] \sim hL^{(0)}h \quad ; \quad L^{(0)}(q,\omega) = 1/(\omega^2 + u_0^2 q^2) \to L(q,\omega) = 1/[\omega^2 + u_0^2 q^2 - \Sigma(q,\omega)]$$

In the lowest order: $\Sigma(q,\omega) = \frac{\mu_0^2 a}{u_0} |\omega|^3$

Conductivity:

$$\sigma(\Omega) = i\frac{e^2}{2\pi\Omega}\left[\frac{u_0^2}{v_F^2}\frac{1}{1+i\Omega\mu_0^2 a/u_0} - \frac{\Omega^2}{6\Delta^2}\right]$$

⇒ **remains ballistic at** $\Omega \to 0$

However

$$\operatorname{Re}\sigma(\Omega) = \frac{e^2}{2\pi}\frac{u_0^2}{v_F^2}\frac{\mu_0^2 a}{u_0} > 0$$

Accounting for anisotropy

$$S_{e-S}[\Psi,\vec{S}] \rightarrow S_{e-S}[\psi,n_z,\alpha] + S_{an}[\alpha]$$

$$S_{e-S}[\psi,n_z,\alpha] = \int d\tau dx \left(\bar{\psi}_{R(\uparrow)},\bar{\psi}_{L(\downarrow)}\right) \begin{bmatrix} \partial_+ - \frac{i}{2}\partial_+\alpha, & \Delta\sqrt{1-n_z^2}(1+\eta e^{2i\alpha}) \\ \Delta\sqrt{1-n_z^2}(1+\eta e^{-2i\alpha}), & \partial_- + \frac{i}{2}\partial_+\alpha \end{bmatrix} \begin{pmatrix} \psi_{R(\uparrow)} \\ \psi_{L(\downarrow)} \end{pmatrix}$$

$$S_{an}[\alpha] = \frac{v_F}{8\pi}\int d\tau dx (\partial_x\alpha)^2$$

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Physical properties of various fields:

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α - massless ("Goldstone") bosons (spin rotations);

n_z - massive ("non-dynamical") boson field → can be integrated out

Collecting things together:

($h(x,\tau)$- source field for electron density)

$$\mathsf{S} \to \mathsf{S}[\psi,\alpha;h] = \mathsf{S}_{e-S}[\psi,\alpha;h] + \mathsf{S}[\alpha;h]$$

Gapped fermions perturbed by boson field

Boson action

$$\mathsf{S}_{e-S}[\psi,\alpha;h] = \int d\tau dx \left(\overline{\psi}_{R(\uparrow)}, \overline{\psi}_{L(\downarrow)}\right) \begin{bmatrix} \partial_+ - \frac{i}{2}\partial_+\alpha + h, & \Delta \\ \Delta, & \partial_- + \frac{i}{2}\partial_+\alpha + h \end{bmatrix} \begin{pmatrix} \psi_{R(\uparrow)} \\ \psi_{L(\downarrow)} \end{pmatrix}$$

Boson action:

$$\mathsf{S}[\alpha;h] = \int d\tau dx \left[\frac{v_F}{8\pi u^2}[(\partial_\tau\alpha)^2 + u^2(\partial_x\alpha)^2] + \mathrm{Re}\left[\varepsilon(x)\,\mathrm{e}^{2i\alpha(x,\tau)}\right] - \frac{1}{2\pi} h\partial_x\alpha \right]$$

(Mapping on Giamarchi-Schulz, 1988)

Here

$$\varepsilon(x) \propto \eta(x)$$

$$u = \frac{J_\parallel}{2\pi}\left(\ln\frac{E_B}{\Delta}\right)^{1/2} << v_F \quad \leftrightarrow \quad K \equiv \frac{4u}{v_F} << 1$$

(range of applicability)

“Canonical form” of the boson action ($K = \frac{4u_0}{v_F} << 1$):

(Giamarchi & Schulz, 1988)

$$S[\alpha] \to \int d\tau dx \left\{ \frac{1}{2\pi K} \left[\frac{1}{u_0} (\partial_\tau \alpha)^2 + u_0 (\partial_x \alpha)^2 \right] + \mathrm{Re}[\varepsilon(x) e^{2i\alpha(x,\tau)}] \right\}$$

Disorder: $\langle \varepsilon^*(x)\varepsilon(x') \rangle = \frac{u_0^2}{\xi_0^3} D_0 \delta(x - x')$, $D_0 = \frac{dv_F}{J} \ln\left(\frac{E_B}{\Delta_0}\right) << 1;$ $d = \frac{<(\delta J)^2>}{J^2}$

Localization at $T = 0$:

RG equation for disorder parameter *D*: $\frac{d D_0}{d \ln l} = (3 - 2K) D_0$

$$D_0(l \to \infty) \to \infty$$

A weak bare disorder grows ⟹ Localization

Summary 1.

Life at $T << \Delta$; qualitative description:

1. Due to specific RKKY, spins tend to align in x-y plane (local order with correlation length $\xi = 4a\, v_F / J_\parallel >> a$)

2. Electrons become "almost gapped". Gap: $\Delta = v_F / \xi = J_\parallel / (4a)$

3. Slow charge transfer is provided by slow variation of spin orientation (azimuthal angle $\alpha(x, \tau)$) described by an effective action $S[\alpha]$.

4. For isotropic electron-spin coupling electron transport is ballistic; for randomly anisotropic couplings electrons are localized at $T = 0$; with a finite resistivity at $T > 0$.
   [Altshuler, Aleiner, Yudson, Phys. Rev. Lett. 111, 086401 (2013)]

Accounting for e-e interactions

A weak repulsive e-e interaction in 3D (2D) – weak effects

(Landau theory of Fermi liquid)

However, even a weak e-e interaction in 1D is "strong"

Fermi liquid → Luttinger liquid

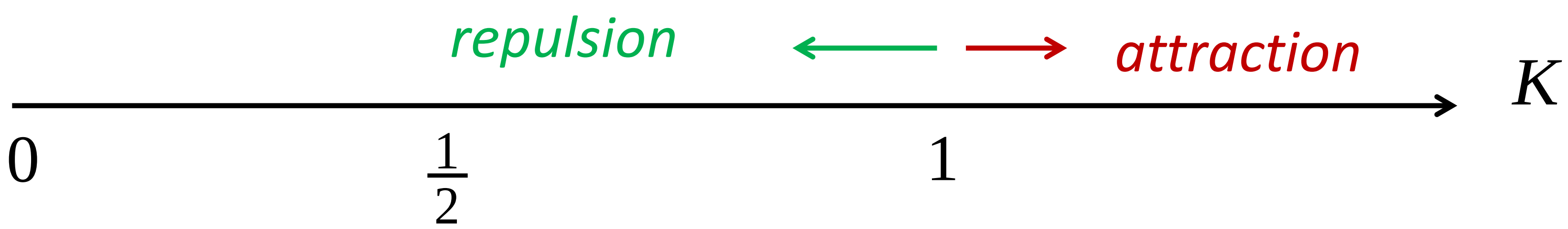
Single-electron excitations are not quasiparticles anymore

Low-energy excitations – collective excitations ("plasmons")

Bosonization $\Psi_{R(L)}(x,\tau) \rightarrow e^{i[\pm\phi(x,\tau)-\theta(x,\tau)]}$

HLL Hamiltonian:

$$H_{HLL} = \frac{u}{2}\int dx\left[\pi K \Pi^2 + \frac{1}{\pi K}(\nabla\phi)^2\right] \qquad [\phi(x),\Pi(x')] = i\delta(x-x')$$



Outline

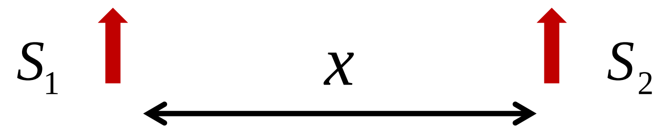
I. RKKY in a Helical Luttinger Liquid (HLL) with weak e-e interaction

II. "RKKY" in HLL with a strong e-e interaction: a paradox

III. Resolution of the paradox (?).

IV. Summary and open questions

Two Spins



$$H_{S-S} \propto -E_{RKKY}(x)[S_1^+ S_2^- + h.c.]$$

$$E_{RKKY}(x) \propto \frac{J_{\parallel}^2}{v_F |x|} > 0$$

(for simplicity $J_z \to 0$)

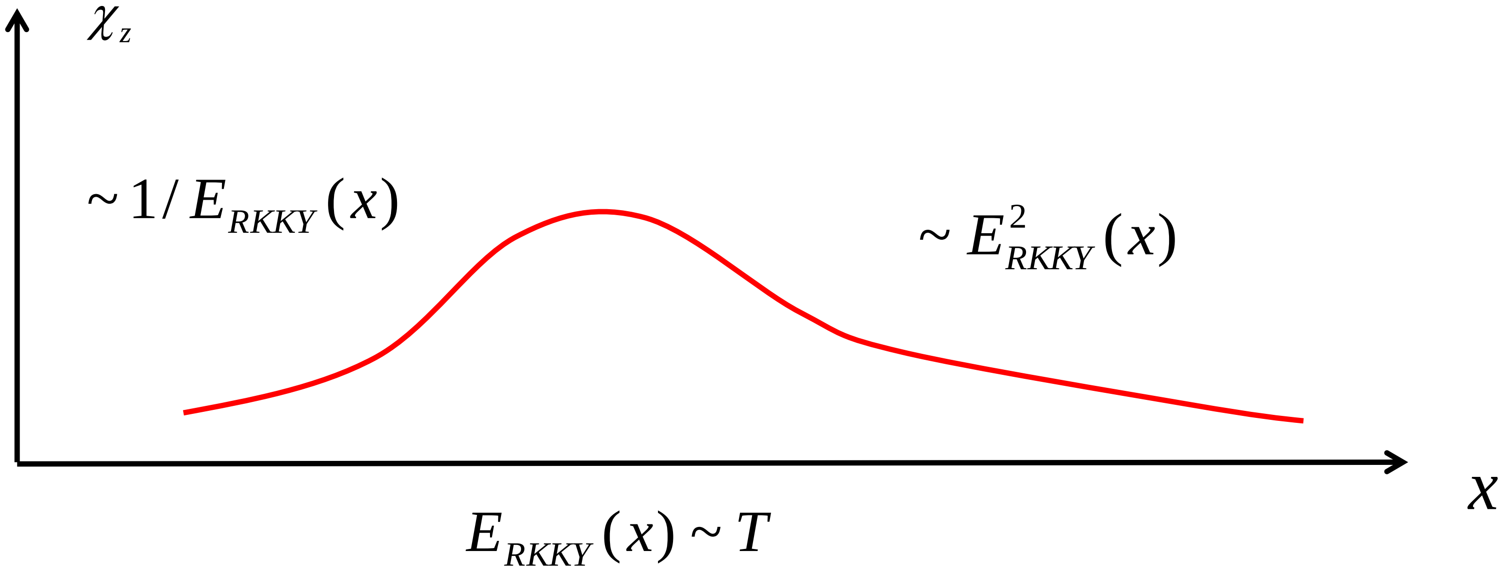
Eigenstates & eigenenergies:

E_{RKKY}	$\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle$
0	$\lvert\uparrow\uparrow\rangle$; $\lvert\downarrow\downarrow\rangle$
$-E_{RKKY}$	$\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle$

Range of interest: $x << L_T = v_F / T$

Susceptibility:

$$\chi_z(2,1) = \frac{\partial M_2}{\partial B_1}\Big|_{B=0} = -\frac{\mu^2}{4}\frac{1}{T}\left[\frac{\sinh(\beta E_{RKKY})}{\beta E_{RKKY}} - 1\right]\frac{1}{\cosh(\beta E_{RKKY}) + 1}$$



Interacting electrons (Helical Luttinger Liquid)

HLL Hamiltonian (bosonization):

$$H_{HLL} = \frac{u}{2}\int dx\left[\pi K \Pi^2 + \frac{1}{\pi K}(\nabla\phi)^2\right] \qquad [\phi(x),\Pi(x')] = i\delta(x-x')$$

Electron-spin interaction:

$$H_{e-S} = -J_z[S_1^z\Pi(x_1) + S_2^z\Pi(x_2)] + \frac{J_\parallel}{2\pi a}[S_1^+ e^{2i\phi(x_1)} + S_2^+ e^{2i\phi(x_2)} + h.c.]$$

Correlation function:

$$\chi(x,\tau) = -\langle T_\tau e^{2i\phi(x,\tau)} e^{-2i\phi(0,0)}\rangle = -\frac{\left(\frac{\pi a}{\beta u}\right)^{2K}}{\left[\sinh\left(\frac{\pi}{\beta}\left(\frac{x}{u}+i\tau\right)\right)\sinh\left(\frac{\pi}{\beta}\left(\frac{x}{u}-i\tau\right)\right)\right]^K}$$

At $T \to 0$

$$\chi(x,\tau) = -\frac{a^{2K}}{[x^2 + u^2\tau^2]^K}$$

Repulsive interaction

$$0 < K < 1$$

strong weak

"RKKY" - formal expansion in $J_{\parallel}^2$ (ignoring for a while J_z terms):

"Weak" interaction

$\frac{1}{2} < K < 1$

At $T \to 0$

$$E_{RKKY}(x) \propto \frac{J_{\parallel}^2}{ua}\left(\frac{a}{x}\right)^{2K-1}$$

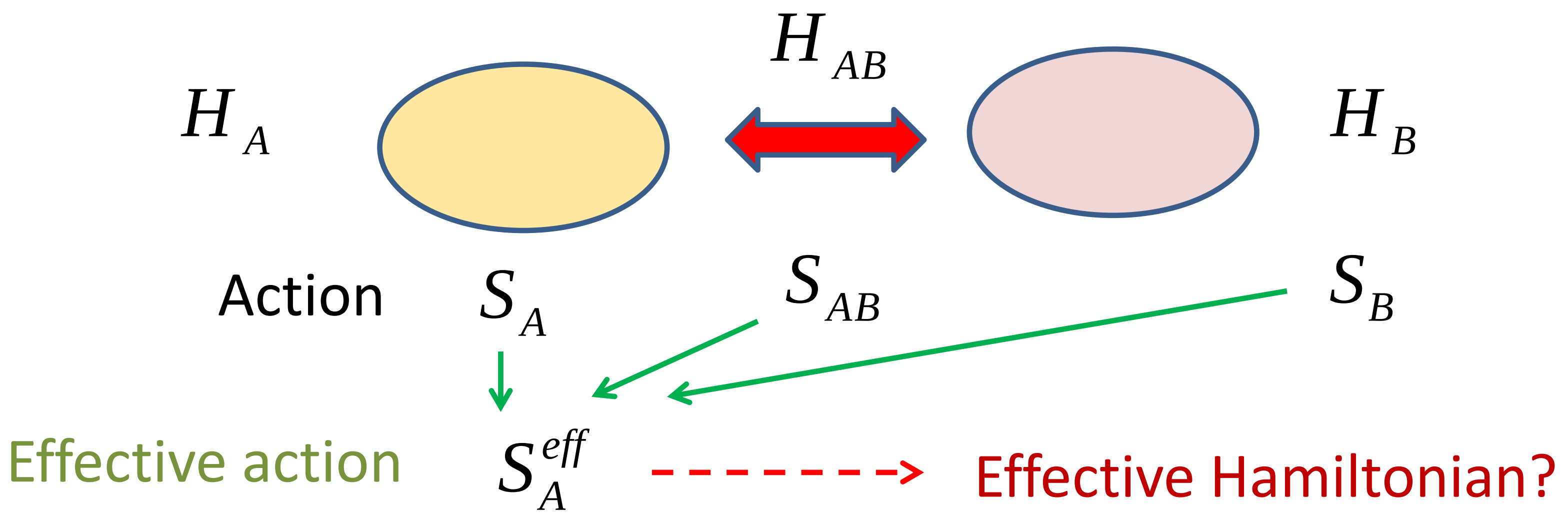
"Strong" interaction

$0 < K < \frac{1}{2}$ Energy grows with distance???

A paradox?

Formal resolution of the “paradox”

1. Limitations of the Hamiltonian description



Our case, **effective** (Matsubara) **spin action** (in $J_{\parallel}^2$ order):

$$S_{S-S}^{eff} \propto J_{\parallel}^2 \iint d\tau_1 d\tau_2 \left[S_1^+(\tau_1)\chi(x,\tau_1-\tau_2)S_2^-(\tau_2) + c.c \right]$$

$$S_j^+ = (\bar{d}_j + d_j)\bar{c}_j, \; S_j^z = \bar{c}_j c_j - 1/2.$$

Restriction to local in time action:

$$(S_{S-S}^{eff})_{loc} \propto J_{\parallel}^2 \int d\tau \left[S_1^+(\tau)\chi(x,i\omega_n=0)S_2^-(\tau) + c.c \right] \equiv \int d\tau\, H_{eff}(\tau)$$

? $\chi(x,i\omega_n=0) = \int d\tau\, \chi(x,\tau)$?

At $T \to 0$

$$\chi(x,i\omega_n=0) \propto \int d\tau \frac{a^{2K}}{[x^2+u^2\tau^2]^K} \propto \left(\frac{a}{x}\right)^{2K-1}, \quad \text{for } K > \frac{1}{2}$$

diverges $\quad \text{for } K < \frac{1}{2}$

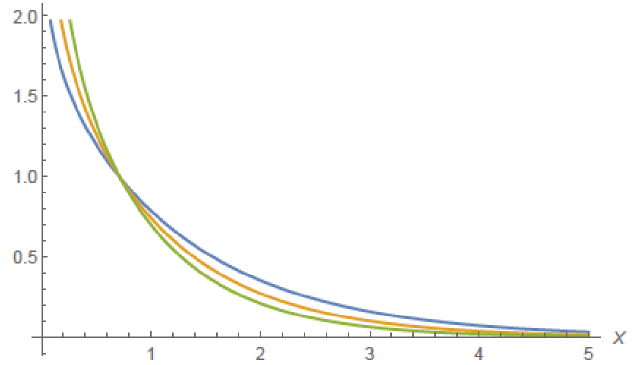
No effective Hamiltonian

Finite temperature, $x << L_T$

$$\chi(x, i\omega_n = 0) \propto \left(\frac{\pi a}{L_T}\right)^{2K-1} \int_0^1 \frac{d\tau}{\left[\sinh\left(\pi\left(\frac{x}{L_T} + i\tau\right)\right)\sinh\left(\pi\left(\frac{x}{L_T} - i\tau\right)\right)\right]^K}$$

$$\propto \left(\frac{a}{x}\right)^{2K-1}, \quad for\ K > \frac{1}{2}$$

smoothly decaying function of $\frac{x}{L_T}$, $\quad$ for $K < \frac{1}{2}$



RKKY vs Kondo:

$$E_{\mathrm{RKKY}} \sim D\,(\rho_0 J_\perp)^2\,(\xi/R)^{2K-1},\; 1/2 < K \le 1;$$

$$T_K \propto \begin{cases} T_K^{(0)},\; 0 < 1-K \ll 1; \\ D\,(\rho_0 J_\perp)^{\frac{1}{1-K}} \gg T_K^{(0)},\; 1-K \gg \rho_0 J_\perp \end{cases}$$

At K< ½ Kondo is stronger than RKKY even at shortest distances

Mini-summary:

No “brute force” Hamiltonian description for $K < \frac{1}{2}$

Correctly reformulated question:

how free energy depends on the distance between spins?

BTW, the above is directly applicable to a system of 1D spinless fermions scattered by two heavy impurities $\Longrightarrow$
the spatial dependence of the induced interaction potential

Two spin problem is much more complicated…

Still, a regular treatment is possible for some particular cases

Treatable cases

Canonical (Emery-Kivelson)

transformation with a unitary operator $U = e^{i\lambda[\phi(x_1)S_1^z + \phi(x_2)S_2^z]}$

$$U\Pi^2(x)U^+ = \Pi^2(x) - 2\lambda[S_1^z\Pi(x_1)\delta(x - x_1) + \Pi(x_2)S_2^z\delta(x - x_2)]$$

$$US_{1(2)}^{\pm}U^+ = S_{1(2)}^{\pm}e^{\pm i\lambda\phi(x_{1(2)})}$$

Modified electron-spin interaction:

$$H_{e-S} = -\tilde{J}_z[S_1^z\Pi(x_1) + S_2^z\Pi(x_2)] + \frac{J_\parallel}{2\pi a}[S_1^+ e^{i(2+\lambda)\phi(x_1)} + S_2^+ e^{i(2+\lambda)\phi(x_2)} + h.c.]$$

$$\tilde{J}_z = J_z + \lambda\pi u K$$

1. Decoupling limit: $\lambda = -2, \quad J_z = 2\pi u K \quad \Rightarrow \tilde{J}_z = 0$

$$H_{e-S} = \frac{J_\parallel}{\pi a}[S_1^x + S_2^x]$$

No energy dependence on the distance between the spin

However, spin-spin correlation functions are not quite trivial:

Expansion in a small deviation from the decoupling limit

$$G_{zz}(\tau) = -\langle T_\tau S_1^z(\tau) S_2^z(0) \rangle$$

$$\delta\tilde{J}_z$$

$$G_{zz}^R(\omega) \simeq i\left(\frac{\pi}{2}\right)^3 (\rho_0 J_z a_{\rm fs})^2 \left(\frac{\Omega_\perp}{\omega_+^2 - \Omega_\perp^2}\right)^2 \frac{\omega}{K} e^{i\frac{R\omega_+}{u}} \rightarrow 0 \text{ at } \omega \rightarrow 0$$

with $\Omega_\perp \equiv J_\perp / 2\pi\xi$.

Compare with the usual RKKY case with $\hat{H}_{\rm RKKY} = -E_{\rm RKKY}(\hat{S}_1^+ \hat{S}_2^- + h.c.)$

$$G_{zz}^R(\omega) = -\frac{\pi}{2} \frac{|E_{\rm RKKY}|}{\omega_+^2 - (2E_{\rm RKKY})^2}$$

Summary 2

1. In a helical Luttinger liquid with two spin impurities, the range of e-S interaction parameters $K < 1/2$ does not allow a brute force Hamiltonian description.

2. There is no paradox of the RKKY energy growing with the distance because there is no RKKY spin Hamiltonian. The system requires description in terms of the free energy.

Yevtushenko & Yudson, arXiv:1709.00325

Thank you!