Novel topologies in superconducting junctions

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Lecture #3

- Semiclassical topological numbers
- Analogy with topological insulators
- Omega-squid
- 4T-ring
- Results Omega-squid
- Results 4T-ring
- General picture: special points
- Vicinity of the special points
- Protection-unprotection transition
- PUT: Landau Hamiltonian

Topology B

- Smile gaps
- Mysteries of smile gap
- Smile gaps in 4T-ring
- Transmission distribution
  of a non-symmetric cavity
- Topology of number of channels
- Explanation of smiling gaps
- 4T-ring: narrow bunch limit

Topology C

Semiclassical topological numbers

- Let’s assume the structure is gapped
- **G** lies at the equator, is the phase μ
- Integral over contour

$$2\pi N_{ij} = \oint d\mathbf{r} \cdot \boldsymbol{\nabla}\mu + \phi_i - \phi_j$$

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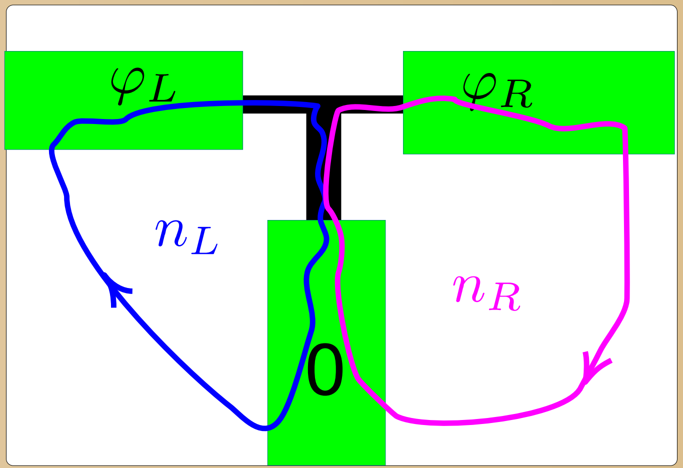
- M terminals → at least M-1 independent numbers

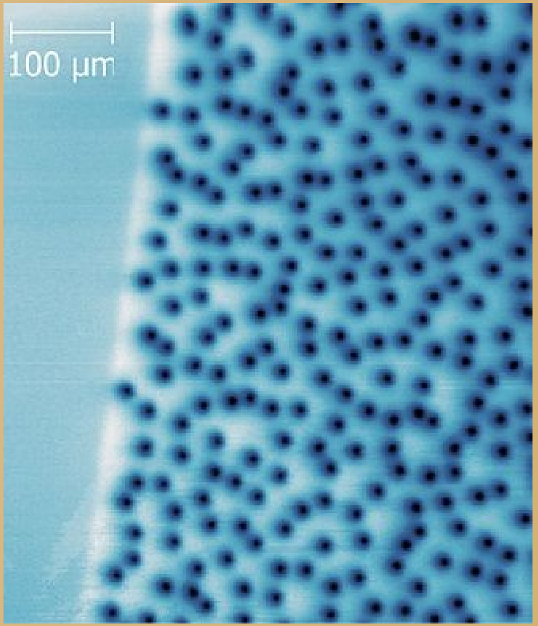


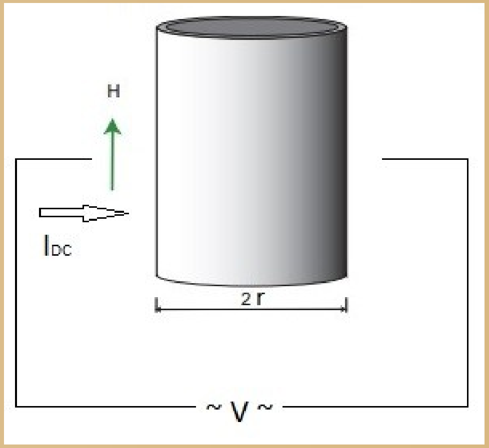
- The numbers distinguish different gapped states

Has nothing to do with flux quantization in superconductors

- μ is no superconducting phase
- For sup. Phase
- Abrikosov vortices
- Quantization of flux in a cylinder
  - Vortex in the hole
- Flux quantization
  - Metastable states
- New topological numbers
  - – well-defined state at each set of parameters

$$2\pi N = \oint d\mathbf{r} \cdot \nabla \phi(\mathbf{r})$$





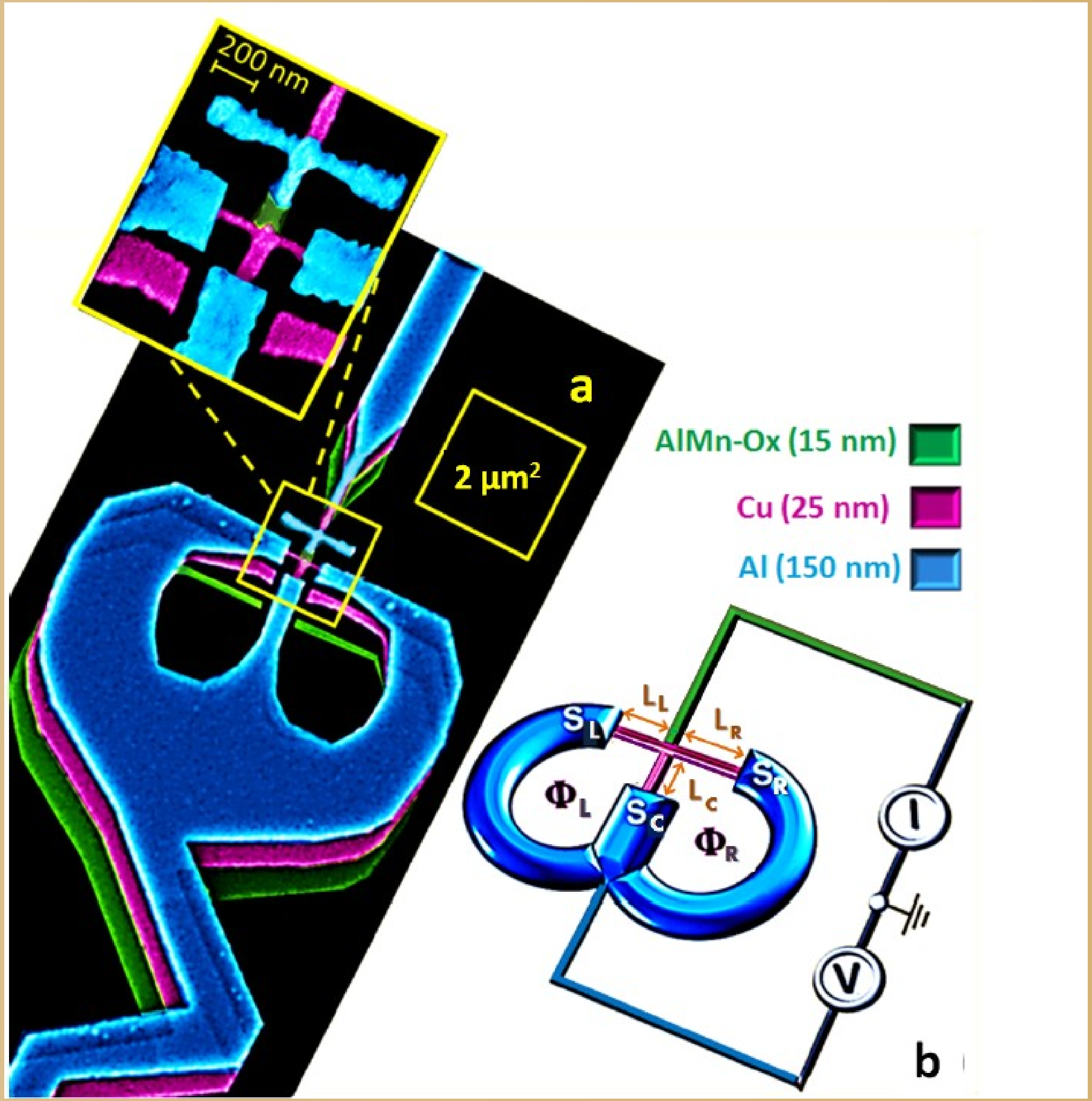
Analogy with topological insulators

- Topologically distinct insulators
- should be **no gap** at the interface
- Separated by gapless states in real space

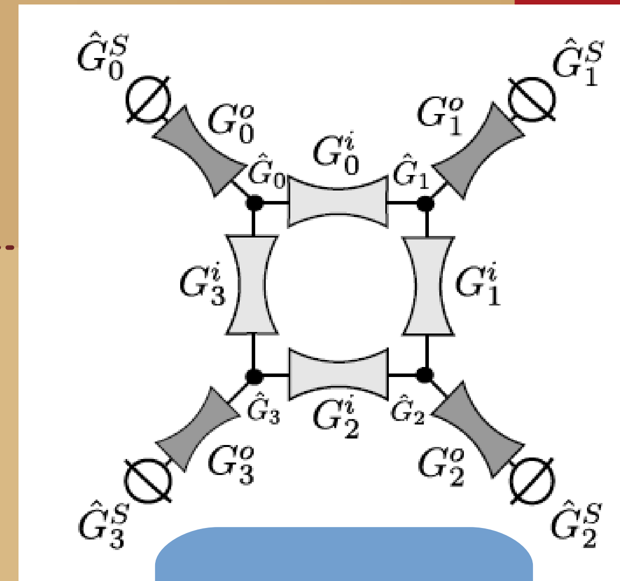
- Topologically distinct gapped phases
- Cannot be changed continously
- Separated by gapless states in parametric space

Omega-squid

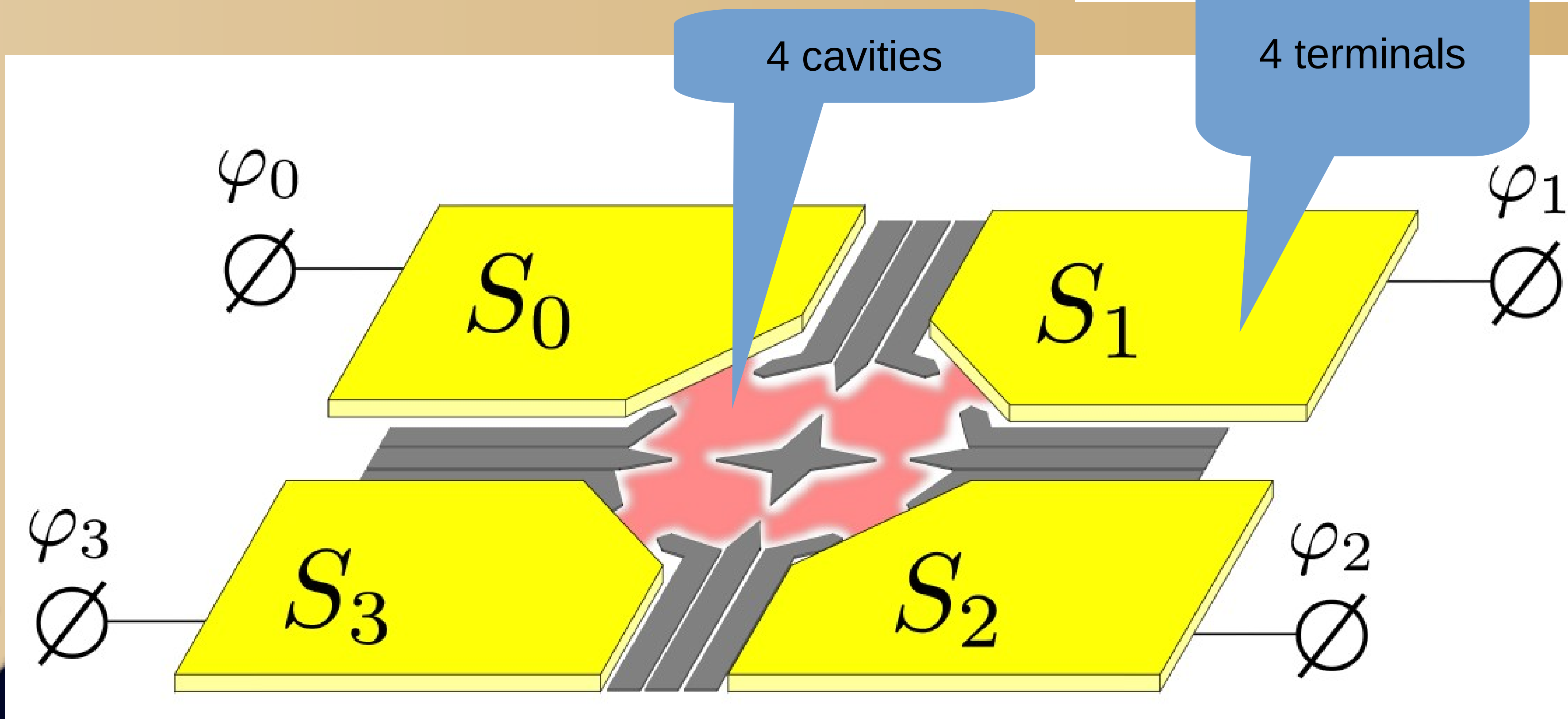
- 3 sup. Terminals
- 1 normal to check the density of states



4T-ring

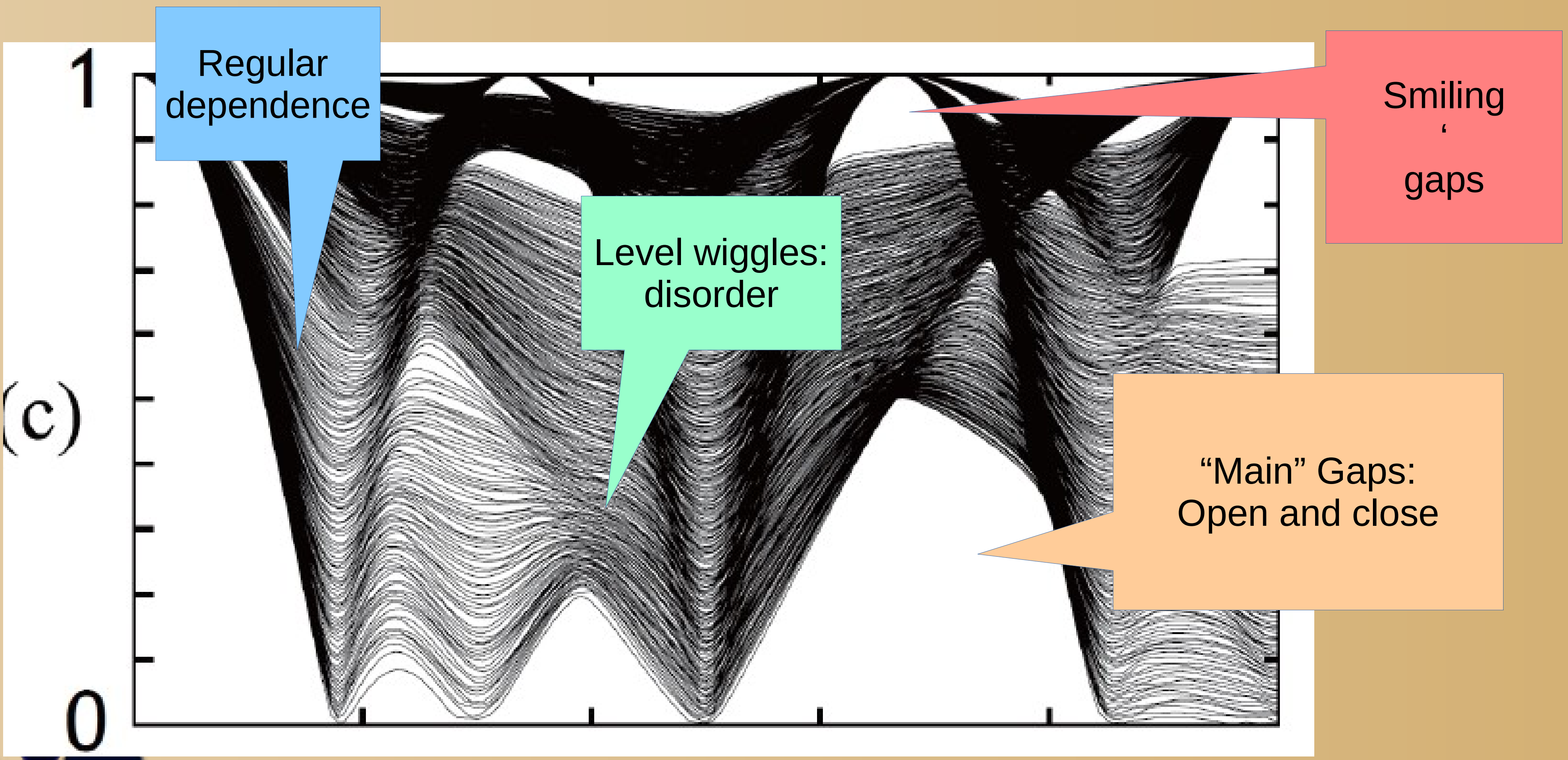


- Ballistic – QPS
- Can be asymmetric, disordered, etc.
- 2d GaAs, graphene

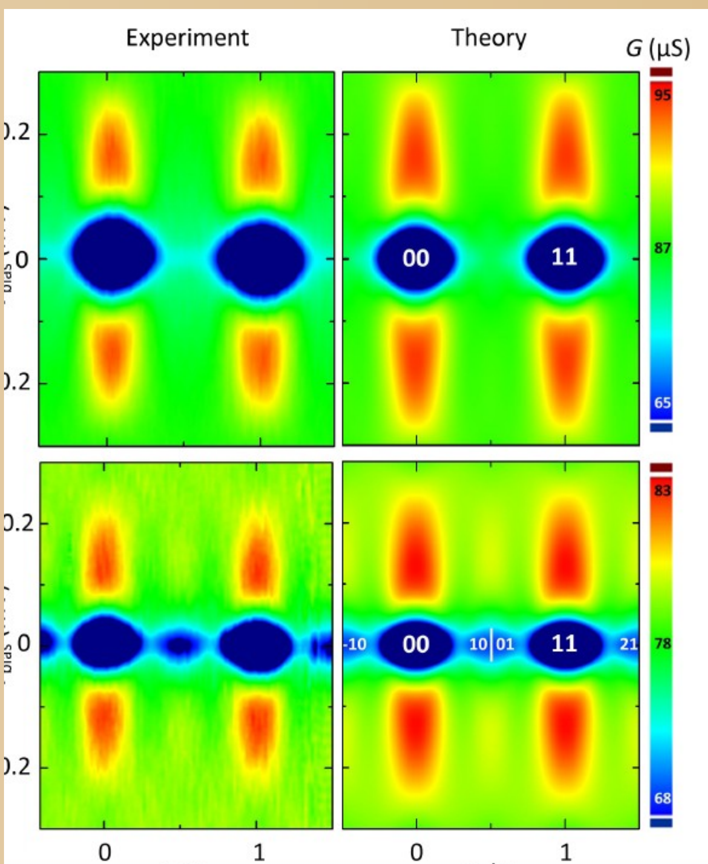


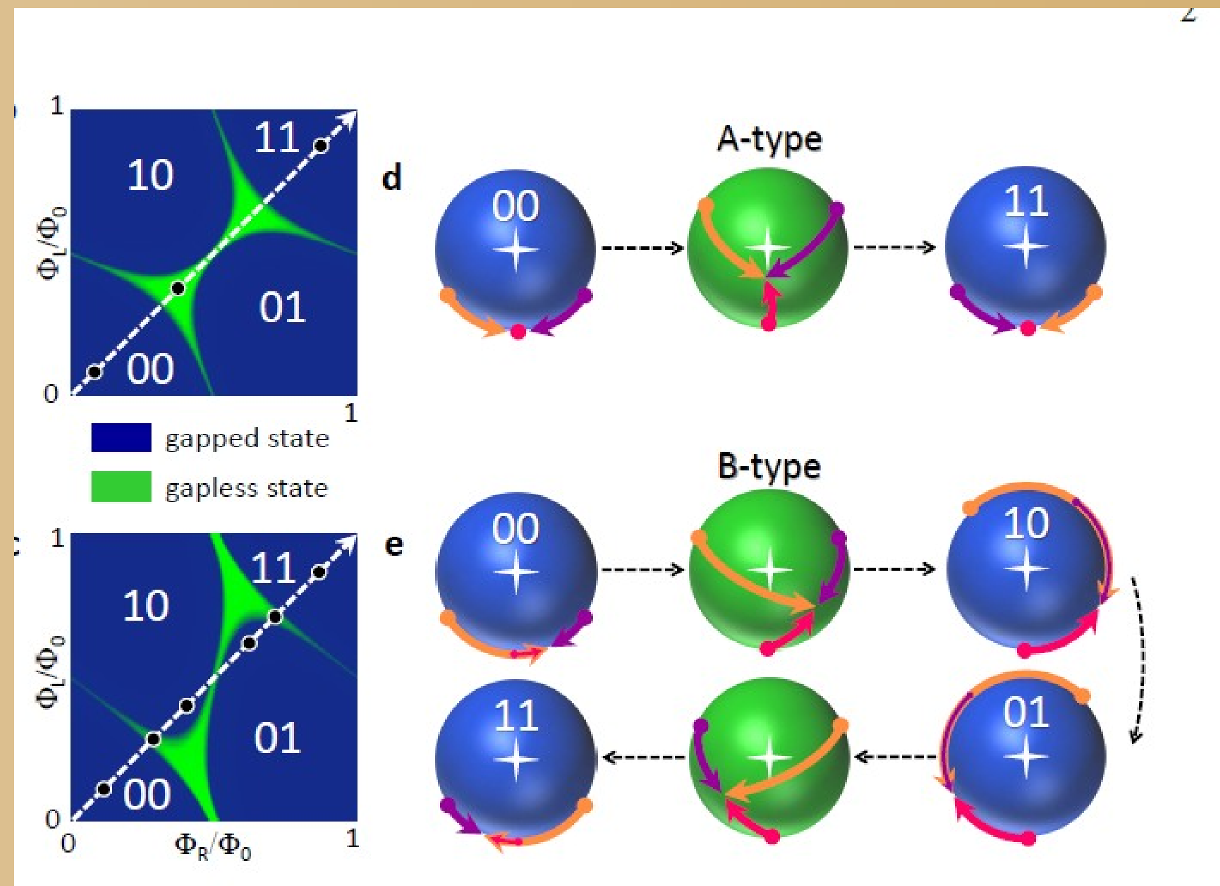
Andreev spectrum of 4T-ring

- Along a line in 3d space of phases $(\varphi_1, \varphi_2, \varphi_3) = (1, 3, 6)\varphi$



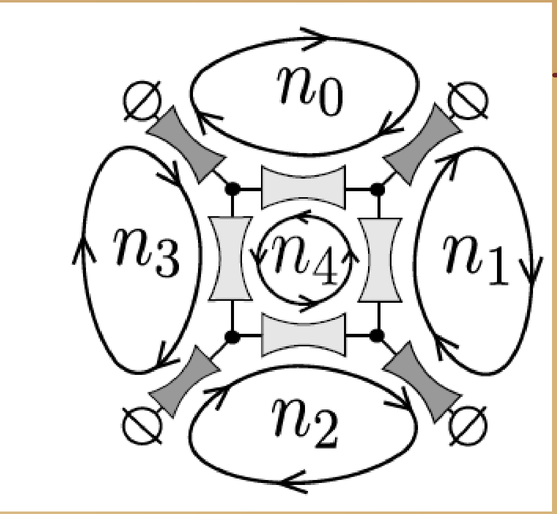
Results Omega-squid

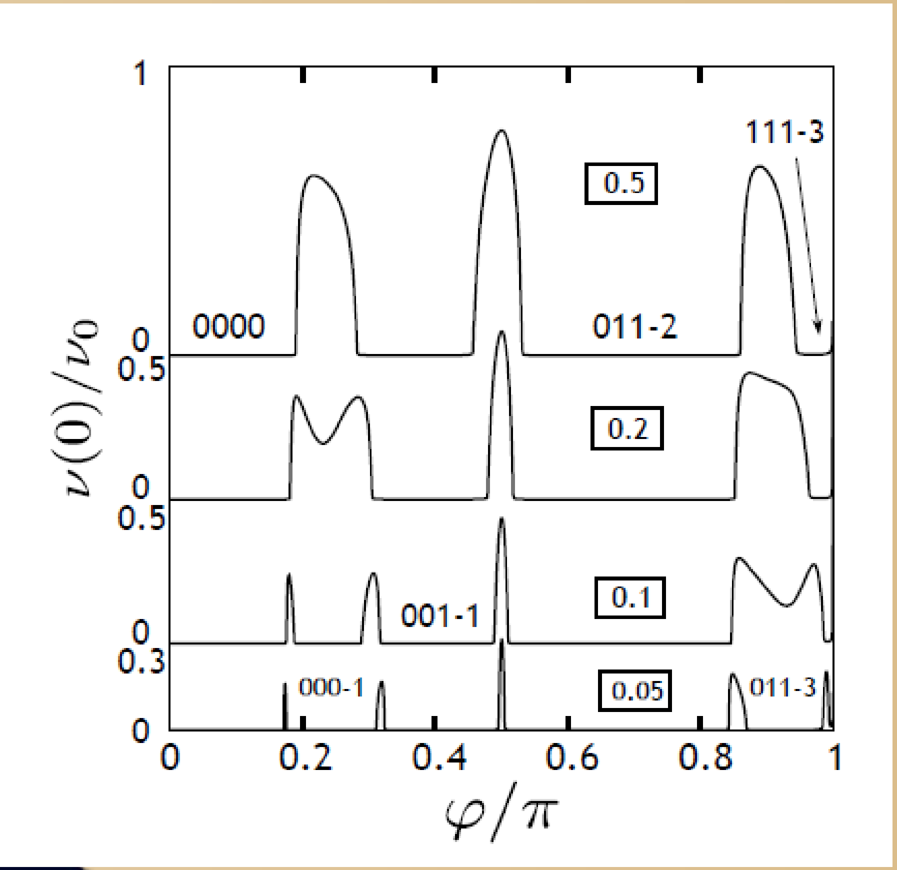


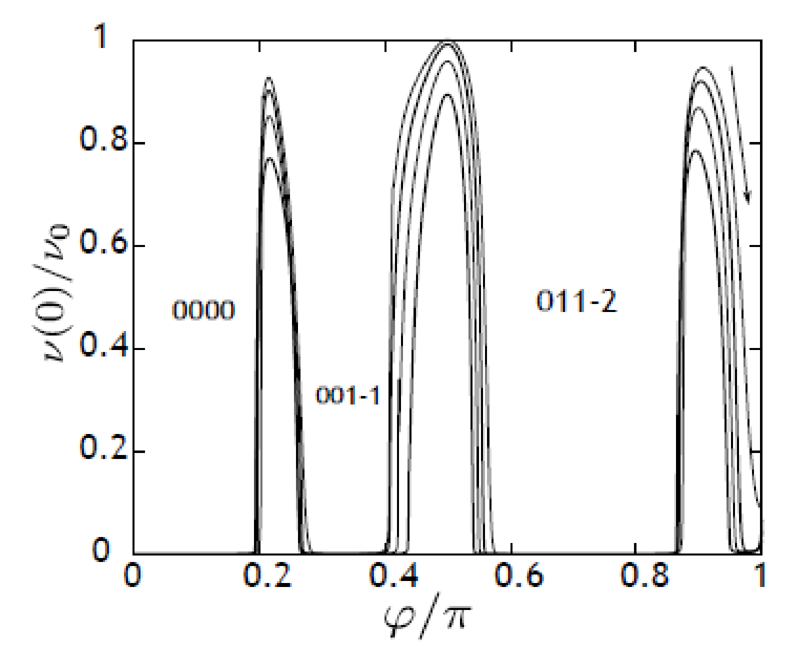


Results 4T-ring

- 4 independent top-numbers
- Gapped states corresponding

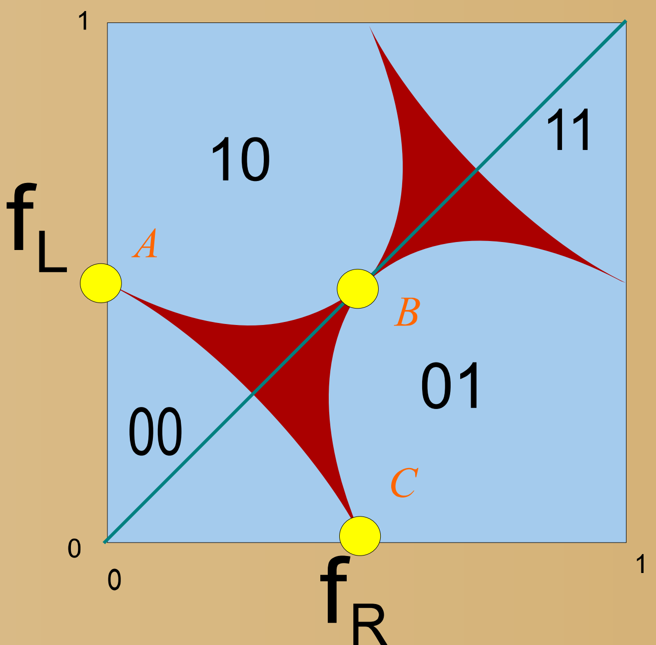






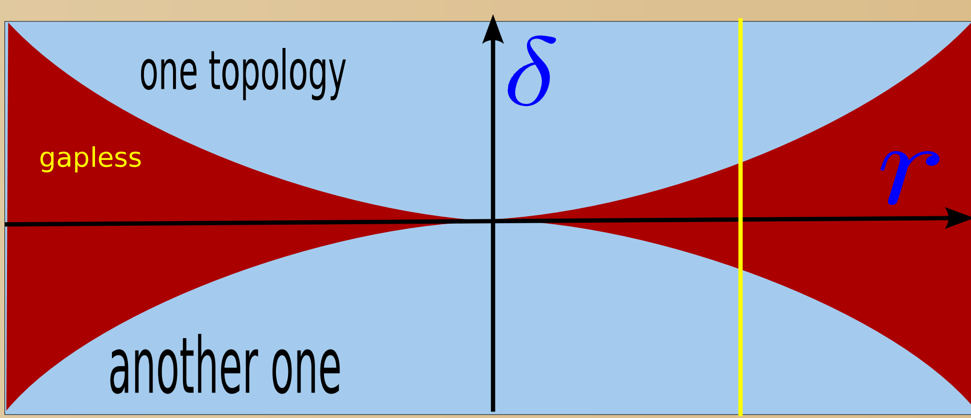
General picture: special points

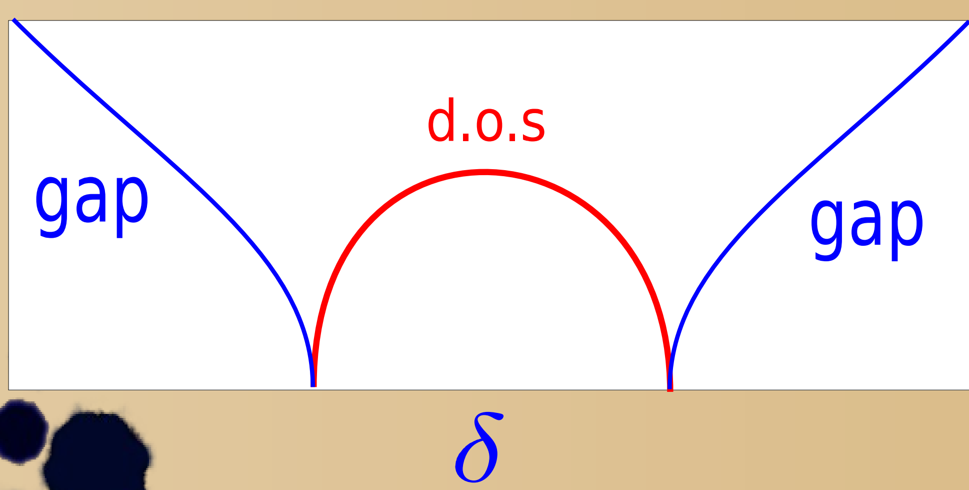
- Special points A,B,C
- Edges of BZ
- Two-terminal system
  - Biased at pi
  -
- Topological protection becomes very thin
- N terminals: $2^{N-1} - 1$ special points

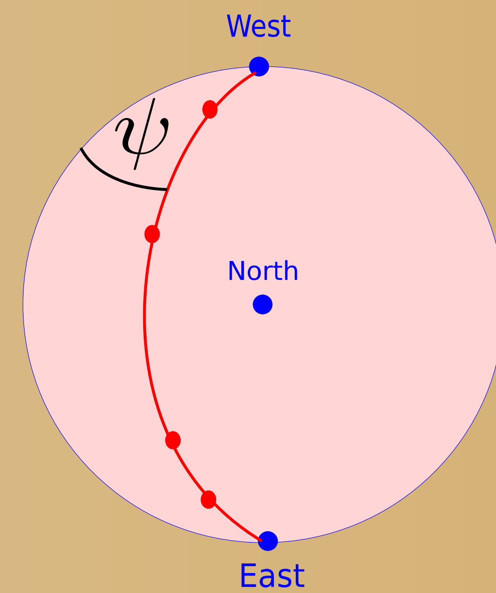


Vicinity of a special point

- Protection near a special point



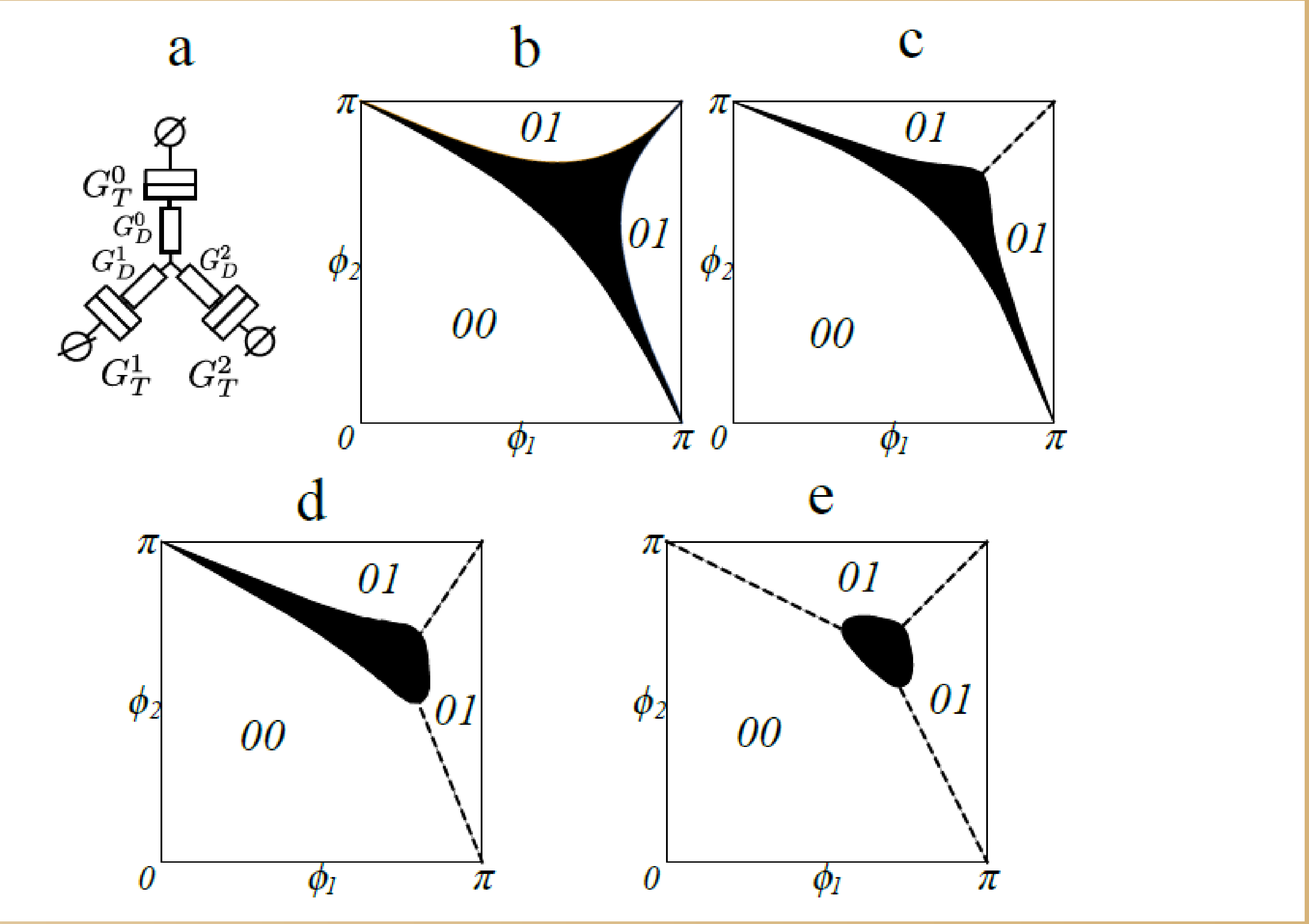




$$\mathcal{S} = \delta\cos\psi + \epsilon\sin\psi + r^2\cos 2\psi$$

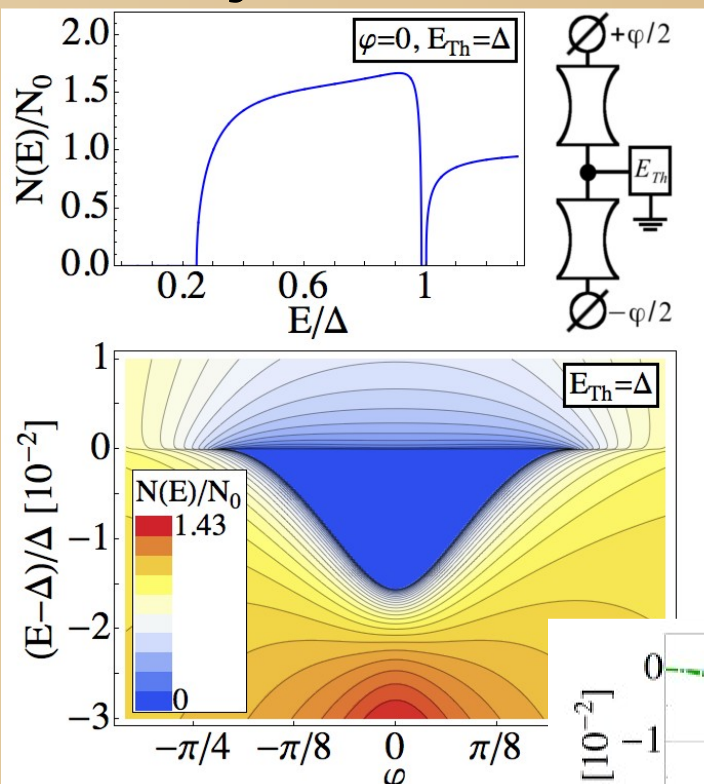
Protection-unprotection transition

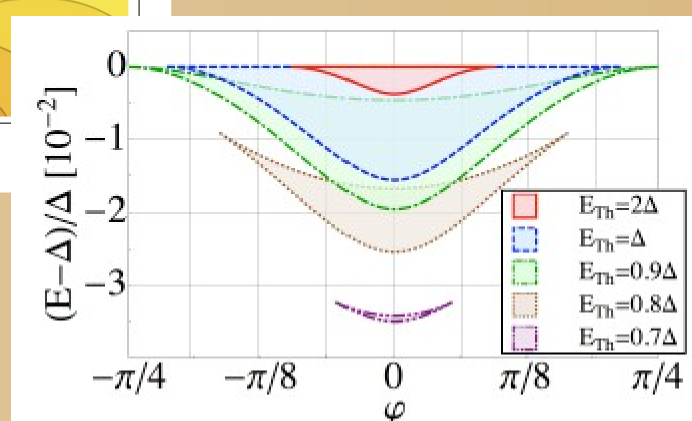
- If continuity is broken (tunnel junctions)



Smile gap

- Phys. Rev. Lett. 112, 067001 (2014)

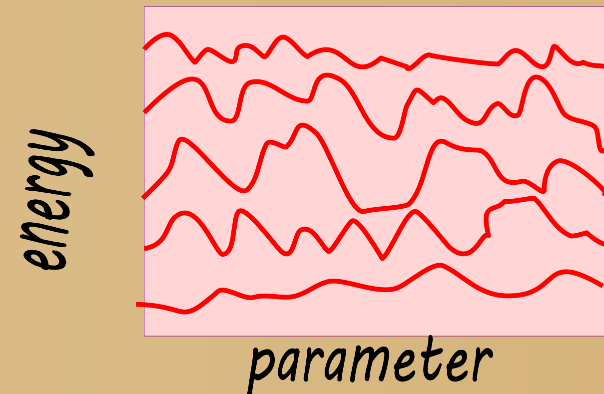


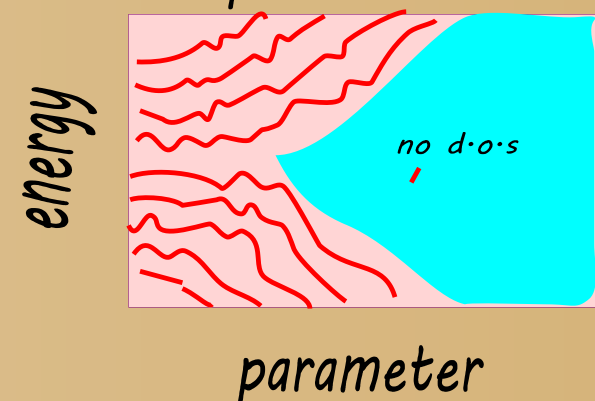


- Quantum semiclassical
- Quantum circuit thy
- d.o.s versus energy and phase
- Secondary gap
  - By the edge
  - Separated from

Mysteries of smile gap

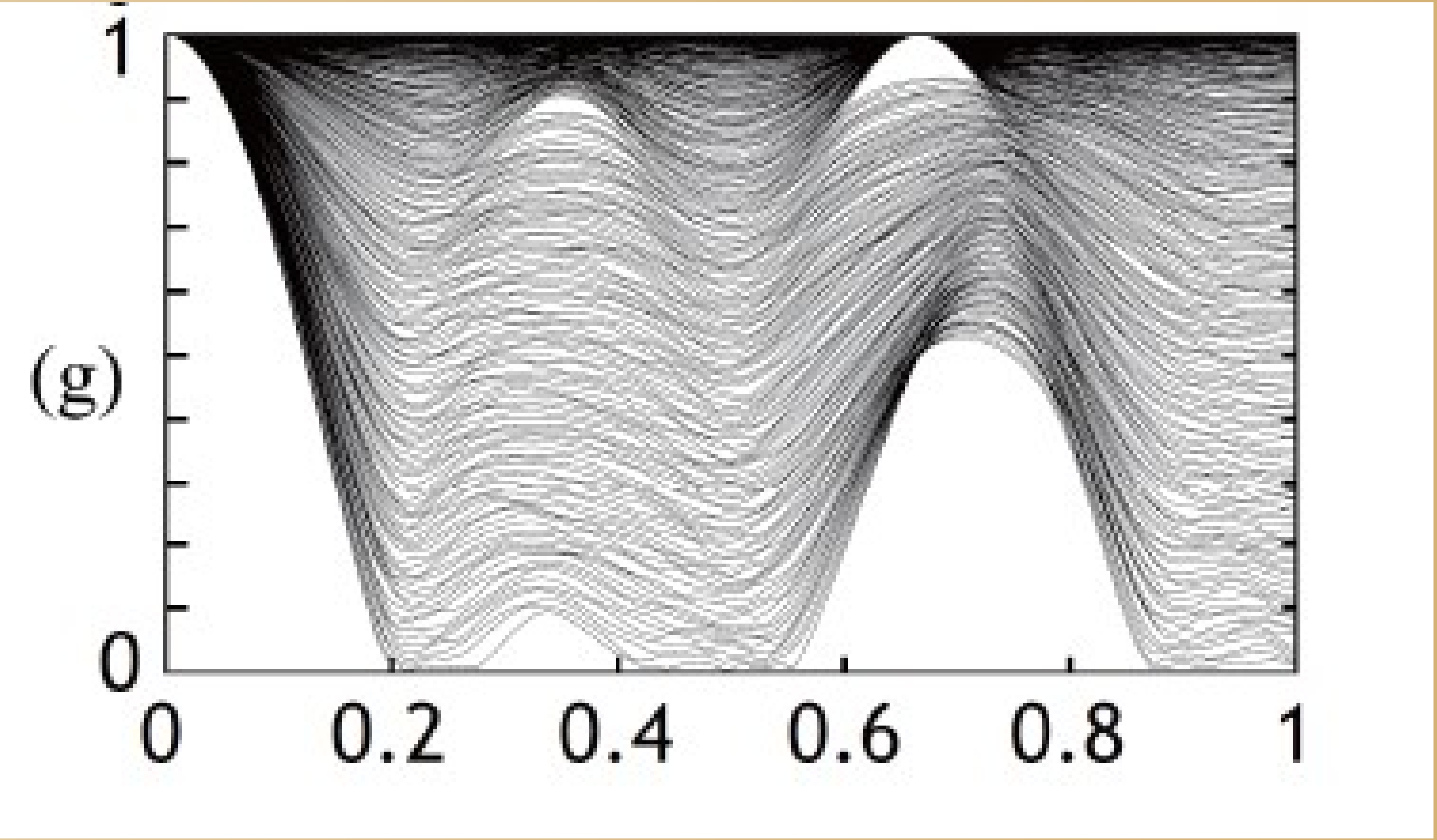
- Semiclassical-continous
- Recall- discrete levels
- In a disordered system
- Democratic system
- Changing parameter = Brownian motion
- And they split – why and how?
- Puzzling problem in general context





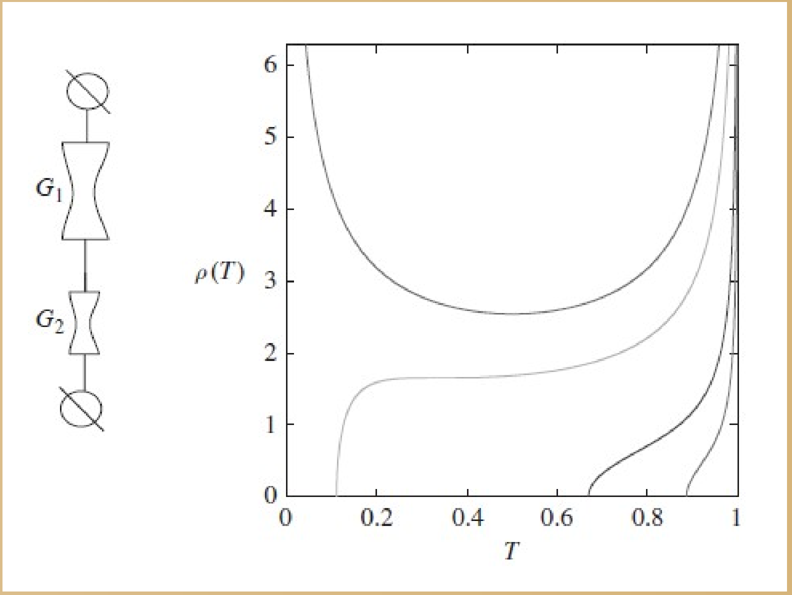
Smile gaps in 4T-ring

- I have a better plot, just wait a min



Transmission distribution of a non-symmetric cavity

- Transmission distribution
- Usually – many channels of low transmission
- Absent for this situation
- Number of channels
  - Topological number!

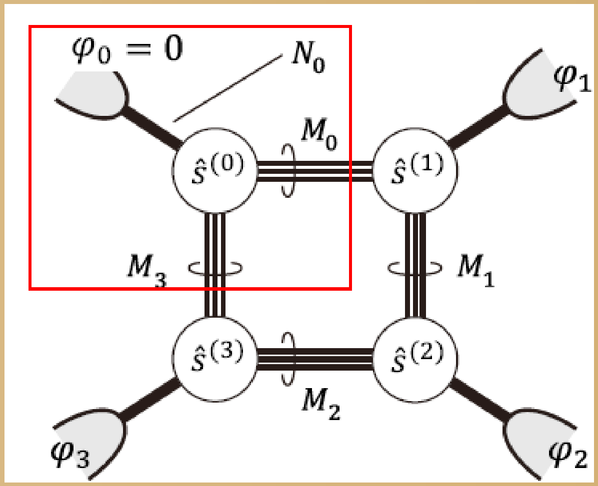


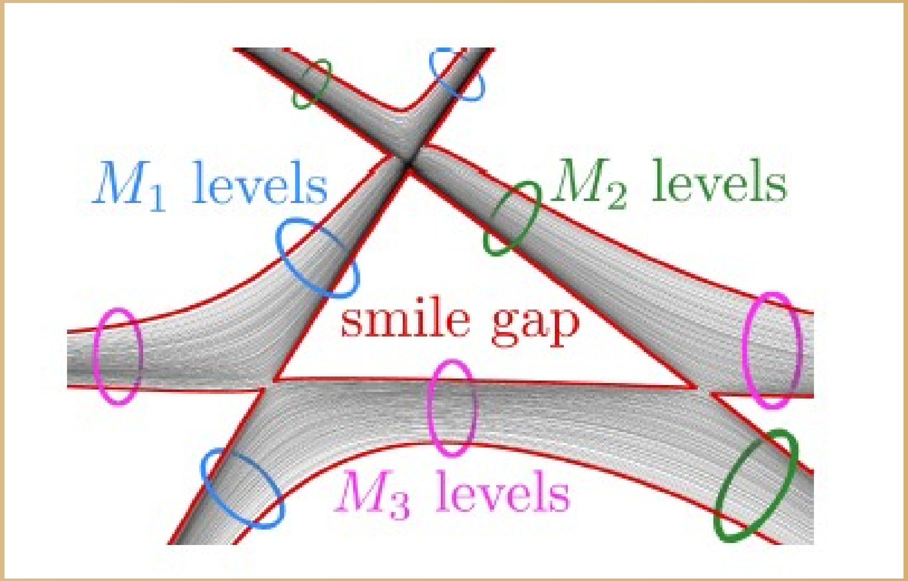
$$\rho(T) = \frac{G_1 + G_2}{\pi G_Q} \frac{1}{T} \sqrt{\frac{T - T_c}{1 - T}}, \quad T > T_c, \quad T_c \equiv \left(\frac{G_1 - G_2}{G_1 + G_2} \right)^2$$

Explanation of smiling gaps

- total number of transport channels robust to disorder in each node
- Differences – number of channels in QPC – robust too
- mode=level? Naive, but works
- There's always $\boldsymbol{M_1}$ (or $\boldsymbol{M_{1,2,3}}$)

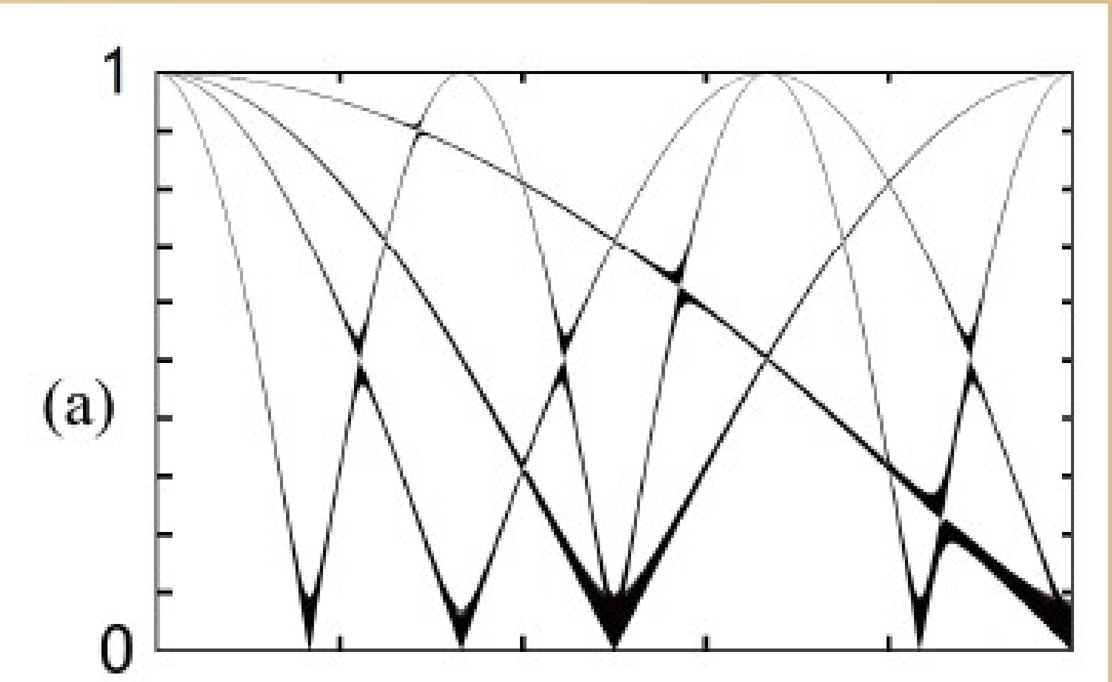
  levels between (smile) gaps,

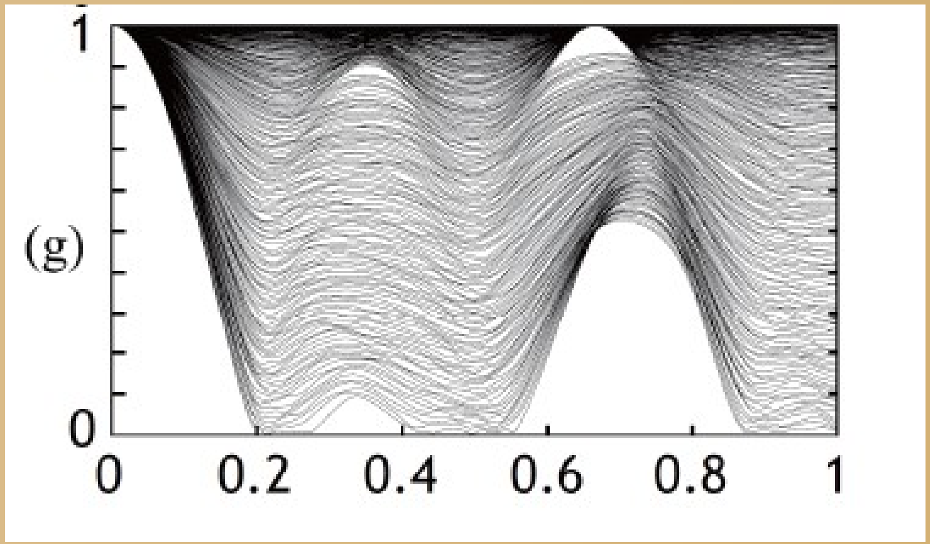




4T-ring: narrow bunch limit

- Parameter: Conductance ratio of outer and inner contacts
- *Open*: The spectum is squized into thin bunches of $\boldsymbol{M_i}$ levels connected to two terminals only, $E = \Delta \cos\left(\frac{\phi_i - \phi_j}{2}\right)$
- *Closed*: spreads over the band (still the gaps)





4Tring: eigenvalue injection

- A single transmission eigenvalue <u>in the gap</u> of the distribution

  = a stray level <u>in the smile gaps</u>

- An interesting opportunity for spectrum manipulation